Chatton on Extension

Introduction

This paper deals with Walter Chatton’s metaphysics of the continuum. Walter Chatton (ca. 1290–1343) was an English Franciscan theologian known for his relentless discussions of Ockham’s philosophical and theological doctrines. He is also counted among nine other philosophers and theologians known for defending an atomist position in the fourteenth century, according to which a continuum is composed of indivisibles. However, Chatton’s atomism has not yet been studied in detail. This paper aims at filling this gap. Its focus will be on spatial magnitudes, as opposed to temporal magnitudes. My aim is to explore


how Chatton articulates three core notions of medieval natural philosophy, namely extension, divisibility, and impenetrability. I will focus on two aspects of Chatton’s position, which are the following.

First, Chatton defends the view that a continuum is composed of a finite number of indivisibles, a position that was usually associated with Democritus’s atomism at the beginning of the fourteenth century. Medieval authors considered Democritus’s atomism the most important alternative to Aristotle’s hylomorphism. Democritus reduces the different kinds of motion that one finds in Aristotle’s physics to local motion, and develops a form of mechanistic physics according to which atoms move in the void and form arbitrary macroscopic objects. Given Chatton’s view, we would expect him to follow Democritus and reject hylomorphism, but this is not the case. Chatton strongly supports the core idea of Aristotelian hylomorphism, namely that substantial changes such as generation and corruption are not reducible to local motion. I will explain why Chatton believes that his redefinition of the concept of a continuum implies that the composition of indivisibles is not the result of their congregation by means of random local motions.

Another interesting feature of Chatton’s atomism is the distinction he draws between the extension of a body and its impenetrability or solidity, i.e. its ability to resist another body moving towards it. The starting point is that extension emerges on indivisibles that congregate in such a way that they form a continuum. Under this assumption, Chatton holds that the possibility of two bodies being in the same place at the same time is not ruled out by the nature of a body. His distinction between extension and impenetrability is driven by theological concerns related to the metaphysics of the Eucharist. Chatton believes that Ockham’s reductionist view on extension, according to which extension is not really distinct from substance or quality, cannot account for the fact that the qualities of the host are at the same place at the same time, although they do not inhere in a substance, given that the substance of the host has disappeared during transubstantiation. Chatton’s criticism helps to distinguish the various properties that Ockham—his main opponent—identified with quantity (namely, infinite divisibility, extension, and solidity) and it reveals that the realist critics of Ockham’s reductionist views on quantity are more diverse and refined than one might expect.

My argument for this point will be in two parts. In the first part, I will lay out the basic principles governing Chatton’s finitist atomism. In the second, I will explain how and why, in

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criticizing Ockham’s view on extension, he distinguishes the extension of a body from its solidity.

1. Atomism

1.1 A Finitist Atomism

Like a handful of other fourteenth-century atomists such as William Crathorn, Gerald of Odo, and John Wyclif, Chatton claims that any given continuum is composed of a finite number of indivisibles immediately next to one another, a claim that has been called “finitist atomism.” His finitist atomism concerns not only geometrical and spatial magnitudes, but also time and motion. According to Chatton, motion is composed of indivisibles immediately next to one another. Similarly, time is composed of instants immediately next to one another.

Finitist atomism about spatial magnitudes is easily refuted by means of geometrical counterarguments based on projection and bi-unique mapping. Unsurprisingly, Chatton’s defense of his finitist atomism comprises a response to the mathematical objections that were popularized by John Duns Scotus. For instance, it can be pointed out that finitist atomism contradicts the possibility of the bisection of every finite straight line as it is presented in Euclid’s Elements, book 1, proposition 10. For if the points constituting a segment are even, the bisection of the segment does not correspond to any point of the segment. If the number of the points is odd, either a point is cut into two, which is impossible, or the cut in the segment

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6 Chatton, Quaestio de continuo, 252: “Dico igitur quod tantum sunt finite res indivisibiles in divisibiles in omni re.” Concerning the authenticity of the text, the editors say (p. 220): “Our first question, if not by Chatton himself, is undeniably one which is secundam doctrinam Chattonis: it follows with remarkable fidelity many of the arguments and conceptions presented in Chatton’s discussion of the composition of continua in Book II of his Commentary on the Sentences.” They believe that the question was written before Chatton’s Sentences commentary, i.e. before 1323.

7 Quaestio de continuo, 250: “motio illa qua movetur et intrat est indivisibilis necessario, ponendo mocionem passivam rem distinctam ab absolutis vel, si sit angelus, propositum, quod pars in spacio correspondet.”

8 Quaestio de continuo, 251: “Inter hoc instans esse, et hoc instans non esse, nullum est instans medium, sed pro eodem instanti non est verum dicere quod hoc instans est et non est, igitur instans immediatum instanti [...].”


corresponds to two points, which is also impossible because there would be no middle indivisible in which to effect the bisection. Chatton grants this, and answers that Euclid’s idea that any segment can be bisected is based on a sensory illusion: only our senses make us believe that the segment is composed of two equal parts.\textsuperscript{11}

Other standard counterarguments have to do with the techniques of parallel and radial projection. For instance, Chatton’s finitist atomism would lead to the claim that the diagonal contains the same number of points as the sides of the square. Let us assume, as Chatton does, that a line is composed of points next to one another, and let us take two points A and B next to one another, and two corresponding points C and D on the opposite side of the square, also next to one another. Let us draw the line from A to C and from B to D. These lines are parallel and both intersect the diagonal of the square at two points. These points must be next to one another. We can repeat the same operation at every point of the lines composing the square. Consequently, the diagonal contains the same number of points as the sides of the square and is not incommensurable with them. In reply, Chatton grants that the diagonal is incommensurable with the sides of the square, but he denies the Euclidean principle that the lines perpendicular to the sides of the square touch the diagonal at only one point. He claims that sometimes the concrete lines that we draw, and not the abstract fictional lines used by the mathematicians, touch the diagonal in two points, depending on the angle at which they touch the diagonal.\textsuperscript{12}

As John E. Murdoch and Aurélien Robert explain, Chatton’s answers introduce physical considerations into mathematics: he resorts to illusions based on the mechanism of sensory perception and to the fact that the points that compose a continuum are not perceptible. Murdoch concludes from this that Chatton “revealed an understanding of mathematics equivalent to that of total incompetence.”\textsuperscript{13} This judgment seems to be shared by Ockham and Wodeham, who do not take seriously his answers to the standard geometrical objections.\textsuperscript{14} They seem little interested in the mechanisms of perception to which Chatton appeals in order to answer the geometrical objections.

\textsuperscript{11} Quaestio de continuo, §103, 262. For a commentary on this passage, see Robert, “Atomisme et géométrie,” 26.

\textsuperscript{12} Quaestio de continuo, §101-102, 261–2; Rep. II, d. 2, q. 3, 131–2. On this subject, see Robert, “Atomisme et géométrie,” 24.


\textsuperscript{14} See for instance Ockham’s and Wodeham’s critique of Chatton’s claim on the incommensurability of the diagonal: Ockham, Quodl. I, q. 9, ed. J.C. Wey, Opera Theologica [OTH] 9 (St. Bonaventure, NY: Franciscan Institute, 1980), 51–6; Adam Wodeham, Tractatus de indivisibilibus, q. 1, a. 1, n. 77, 82–4. For Wodeham’s
What is interesting, however, is not that Ockham and Wodeham criticize Chatton on geometrical grounds, but that their critiques go beyond mathematics. Chatton’s indivisibles are not mere unextended points: they have a nature. As Robert has shown, Chatton’s finitist atomism is part of a positive metaphysical program whose main motivation is to deny the possibility of a created infinite, a motivation to which I now turn.  

1.2 Against Infinitism

In book III of the *Physics*, Aristotle denies the possibility of an actual infinite and admits only the potential infinite that was associated with the infinite divisibility of the continuum. As Aristotle puts it, any permissible infinite is not that beyond which there is nothing, but rather that beyond which there is always something, which corresponds to what medieval authors would call the infinite *in potentia*. Medieval authors rephrased Aristotle’s characterization by saying that the rejected actual infinite is a quantity so great that it could not be greater (*tantum quod non maius*), while the permissible potential infinite is a quantity that is not so great that it could not be greater (*non tantum quin maius*).

The majority of Chatton’s contemporaries agree with Aristotle that the continuum is infinitely divisible, in the sense that the number of divisions (or the number of parts that would result from this division) is only potentially infinite, i.e. such that the process of dividing the continuum can always be reiterated, and not actually infinite, i.e. such that the divisions of the continuum can be done all at once. Chatton believes that the distinction between the potential and the actual infinite cannot be applied to the composition of spatial magnitudes. For him, if spatial magnitudes are infinitely divisible, they are composed of an actual infinity of points. Continua can be of different magnitudes, since a part of a continuum is as infinitely divisible as the continuum taken as a whole. Therefore, the philosophers who accept that the continuum is infinitely divisible grant that some infinities are greater than others and that the former can contain the latter as their parts. But, as Murdoch explains, “if...
one allows the existence of actual infinites, some infinites will be greater than other infinites which are equally parts of the former; but it is axiomatic that all actual infinites are equal; therefore, in this instance a part is not less than, but equal to, its whole.”

This runs counter to the Euclidean axiom that a whole is greater than its part.

At bottom, Chatton’s attitude amounts to regarding unequal infinites, one of them being a part of another, as strictly impossible, and maintaining therefore that one had to reject as equally impossible the postulate that gave rise to the paradox, namely that a continuum is infinitely divisible. Thus, Chatton rejects the new idea, popularized by Henry of Harclay, that one infinite can be greater than another: contrary to William of Ockham and Gregory of Rimini after him, he reveals a conservative attitude to the innovations of his predecessors.

Moreover, he believes that any defender of the infinite divisibility of the continuum accepts that the division of the continuum can be done all at once, which was highly debatable at his time.

Chatton illustrates his argument with several examples. For instance, in the divisibilists’ view any continuum such as a stone would have as many parts as the heavens, or even more, which is absurd. More serious still, in this case the parts of the stone would be infinitely many, and thus the quantity of the stone could possibly exceed even God, which is not acceptable.


20 Rep. III, d. 13, q. 2, 90: “Intellectus aliorum includit contradictionem, dicere aliquid esse infinitum, quod scilicet includat non tot partes quin plures. Probo per argumentum supra de parte ablata, nam tunc totum esset aequale part.”


22 On this question, see Norman Kretzmann, “Adam Wodeham’s Anti-Aristotelian Anti-Atomism,” History of Philosophy Quarterly 1 (1984), 381–98; Edith Sylla, “God, Indivisibles, and Logic in the Later Middle Ages,” 84; [AUTHOR].

23 Rep. II, d. 2, q. 3, 137: “Praeterea, alter lapillus excederet tot creaturas quot Deus; habebit enim tot partes quot caelum, immo quot infiniti caeli si essent. Aut enim est dare maximam multitudinem partium eius, et propositum; aut non, et tunc quapropter utrum partes caeli non excedant multitudinem partium lapilli, et iterum propositum; aut excedunt, et tunc si supraput illa in quibus est excessus sive finita sive infinita, residua erunt tot quod non plura, et iterum propositum. Unde contradictio mihi videtur quod ista multitudo excedatur ab illa et
Chatton’s view is driven by the idea that only God is infinite. There is no actual
infinite in nature, i.e. an infinite whole whose parts can be taken together at the same time. The only form of acceptable infinite in nature is a series of successive things in an endless
succession, such that it is impossible to take all its past, present, and future parts together. When Aristotle says that the continuum is infinitely divisible, he means that this is possible
only in the imagination of the mathematician. This is confirmed by the way Aristotle is read
by Averroes, who claims that the division of the continuum is infinite, but only according to
an imagination which is the one of “impossible things” (*tantum secundum imaginacionem quae est impossibilium*).

1.3 Against Democritus

Chatton’s rejection of the existence of actual infinites in nature might explain why he
chooses to defend finitism, but it does not explain what kind of finitism he adopts. Its nature is
determined by his opposition to Democritean atomism. Democritean atomism is characterized
by several theses, including the claims that there are atoms that are endowed with a
homogeneous and impenetrable magnitude and that are ingenerable and incorruptible; that
there is empty space; that all bodies are composed solely of atoms and their aggregates; and
finally, that causation among bodies is limited to collisions among atoms and their
aggregates. The last claim can be seen as an expression of mechanism. Chatton is strongly
opposed to such a view. He defends the Aristotelian idea that substantial changes, i.e. the

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24 *Rep.* I, d. 2, q. 4, a. 3, 171–2: “Et ad dubium huius articuli, cum quaeritur utrum primum ens sit infinitum, dico quod ‘infinitum’ adduc sumitur dupliciter: vel quod esset nobilium omnibus simul sumptis, vel quolibet illorum
divisim. Si secundo modo […] potest sufficienter persuaderi. Si primo modo, non, sed tenetur fide tantum.” See also *Rep.* I, d. 3, q. 1, 309: “Non est evidens infinitum esse nisi fide, igitur nec Deum esse. […] infinitum non
cadit sub iudicio humano.” For a commentary on these passages, see Robert, “Atomisme et géométrie,” 29–30.
25 *Rep.* II, d. 2, q. 3, 137: “Et cum dicis quod continuum est divisibile in infinitum, istud maxime debereet movere
contra me, sed dico quod non valet, quia ex quo opinio contraria negans indivisibili vel ponens ills etiam dicit
quod hoc est tantum secundum imaginacionem, ita dicam ego quod hoc est secundum imaginacionem
mathematicam tantum.”
26 *Quaestio de continuo*, §37, 241: “Sexto: commentator, 5 Physicorum, continuum est divisibile in infinitum,
dicit quod hoc est tantum secundum imaginacionem quae est impossibilium.”
27 This list comes from Robert Pasnau, *Metaphysical Themes, 1274–1671* (Oxford: Oxford University Press,
2011), 91. Pasnau goes on to say that “Democritus himself may have endorsed all of these theses, but it is hard to
find any subsequent atomist who went so far” (ibid.). For a detailed presentation of Democritean atomism in
medieval and early modern philosophy, see Christoph Lüthy, John E. Murdoch, and William R. Newman,
“Introduction: Corpuscles, Atoms, Particles and Minima,” in Lüthy, Murdoch, and Newman (eds.), *Late

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generation and corruption of substances, are not reducible to the aggregation of atoms by local motion.  

Chatton believes that Democritus’s mechanism is intimately connected to his claim that atoms exist in act in the continuum: they are not only ontologically prior to the whole continuum, but also independent of each other in such a way that their aggregation by local motion is completely arbitrary. This implies, Chatton believes, that Democritean atoms are merely contiguous to each other. Continuity would never emerge on their juxtaposition.

This last claim finds its justification in Chatton’s discussion of Aristotle’s critique of Democritus’s atomism in book VI of the Physics. Aristotle argued that indivisibles cannot touch. Any two things in contact must either touch part to part, part to whole, or whole to whole. But the first two possibilities must be rejected, since indivisibles have no parts. If the third possibility were admitted, the two could not constitute a continuum of a size greater than a sole indivisible." Of course, an atomist will want to claim that Aristotle has no grounds for the conclusion he derives from the hypothesis that two atoms touch whole to whole. However, Chatton is convinced that Democritus has no answer to Aristotle’s criticism. Indeed, his atoms form aggregates only randomly. This implies, Chatton believes, that there is no natural cause in virtue of which they could be distributed in one spatial region or another, which means that nothing prevents Democritus’s atoms from being superimposed on one another. As a consequence, Aristotle is right in saying that two Democritean atoms added to each other “do not produce something bigger.”


30 Rep. II, d. 2, q. 3, 126: “Dico quod continuum componi ex talibus in actu includat contradictionem, quia eo ipso quod continuit et contiguum different oportet quod partes continui uniantur et faciant per se unum, quod si non, non facerent continuum sed contigua essent tantum.”

31 Aristotle, Physics VI.1, 231a26–b6.

32 Rep. II, d. 2, q. 3, 125: “Quid intelligis per indivisibile? Aut aliquid tale quod, additum alteri, non faceret mius et sic est contradictio quantum componi ex indivisibilibus; vel intelligis per indivisibile aliquid tale quod non habet partes intrinsecas, additum tamen alteri facit maius, et sic quantum componitur ex indivisibilibus.” For a more detailed presentation of Aristotle’s “impossibility of contact” argument, see Murdoch, “Superposition, Congruence and Continuity.” 422.

33 The criterion for superposition is Euclid’s axiom of congruence, according to which “things which coincide with one another are equal to one another.” See Thomas Heath, The Thirteen Books of Euclid’s Elements (repr. New York: Dover, 1956), vol. 1, 155; for commentary on this axiom, see Murdoch, “Superposition,” 417.

34 Rep. II, d. 2, q. 3, 130–31: “[...] tunc totum tangeret totum et esset in eodem situ nec facerent mius, illud tantum vadit contra Democritum qui ponit congregationem indivisibilium quorum unum per naturam natum est esse in eodem situ cum alio.” Rep. II, d. 2, q. 3, 126: “Item, aliqua ratio eius currens ad illum intellectum est quod indivisible additum indivisibili non facit mius. Haec est negativa et potest verificari dubius de causis: vel scilicet quia non constituent aliquid per se unum additum alteri non facit mius, et hoc directe est contra Democritum. Alius intellectus est quod ex quo secundum Democritum potuerunt indifferenter congregari ad communem situm et non haberent incomposibilitatem ad essendum per causas naturales in eodem situ; ergo non habeant ex naturis suis quod distarent nec per consequens quod quantum posset ex eis componi, quia quantitas
To avoid having his atomism fall prey to the same difficulty, Chatton relies on Harclay’s solution to Aristotle’s paradox of touching, which is that indivisibles do touch whole to whole but “with respect to distinct locations” (secundum distinctos situs). Two points can touch if they are situated in contiguous places.35 Like Harclay, Chatton starts with Aristotle’s definition of succession in Physics V, chapter 3, 36 and claims that two indivisibles are successive when there is no other thing of the same nature between them. This would not preclude there being between them something of a different nature, such as a gap for instance, under the assumption that void exists, or an indivisible of another nature. So, indivisibles of the same nature can occupy different spatial locations (situs) that are successive to each other, and they can touch each other without being contiguous in the Aristotelian sense, i.e. in the sense that there is nothing between them, either of the same nature or of a different nature.37 This explains why Chatton affirms that indivisibles have a nature in virtue of which it is impossible for them to be in the same place at the same time. Two indivisibles do not superimpose when they are in contact with each other: they are impenetrable.38

Given these conditions, how can a finite number of atoms form a continuous whole? Chatton claims that a continuous whole is a whole whose parts are indivisibles of the same nature (ratio) and successive to each other. By contrast, indivisibles that are successive to each other without being of the same nature (ratio) do not form a unit; they are not continuous but only contiguous.39 Thus, the mere juxtaposition of a plurality of adjacent indivisibles is
not sufficient for them to form a continuous magnitude. It is also necessary that these adjacent indivisibles be of the same nature. Continuity is not reducible to mere contiguity. Indeed, it is in virtue of an essential property of a given indivisible that, when joined to another indivisible, it forms a continuous magnitude.

Consequently, it is in the nature of an indivisible to be part of a continuum under normal conditions. In Chatton’s words, an indivisible exists in potency in the continuum, not because it has a special mode of existence in the continuum, but because it is not separated from the other parts of the continuum to which it naturally belongs. Indeed, the parts of the continuum exist and are really one as much as any other thing or entity is one. Chatton thus successfully avoids reducing continuity to mere contiguity, as did Democritus on his view.

Chatton’s indivisibles are not like those of Democritus, whose atoms have size, shape, and motion. Chatton’s indivisibles are extensionless, but occupy a spatial position and are naturally impenetrable. The ontological primacy of the part, however crucial in understanding Chatton’s atomism, does not imply any form of Democritean atomism according to which all the atoms first exist separately.

For indivisibles to form a continuum, it is necessary that they be of the same nature. This explains why only integral parts (i.e. parts of the same nature) form a continuum, not

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40 *Quaestio de continuo*, §53, 244: “[…] quid intelligo per indivisibile in potentia? Illud quod non habet plures partes eiusdem rationis, tamen facit per se unum et continuatur alteri parti eiusdem rationis, et non per se existit in esse separato ab alia parte; posse, tamen, per aliquam potentiam sic per se existere, et sic, tunc esset indivisible in actu.” See also *Quaestio de continuo*, §10, 236.

41 *Quaestio de continuo*, §54, 245: “Per esse partem intelligo unam rem sic se habere ad aliam eiusdem rei quod vere ex istis resultat continuum distinctum a rebus contiguatis quibuscumque, ad intellectum Aristotelis.”

42 *Rep.* II, d. 2, q. 3, 123: “[…] quod pars est in potentia in toto, id est, non est per se et separatum ab aliis dum est pars, non quin quando est in toto sit in rerum natura extra animam; est ergo pars in potentia id est, non est in actu separato non faciens per se unum cum alio.” *Rep.* II, d. 2, q. 3, 125: “[continuum] non componitur ex indivisibilibus in actu quae non faciunt per se unum modo praeeposito, cuius oppositum dicebat Democritus contra quem arguit Aristoteles I de generatione cap. 7, qui posuit continuum componi ex indivisibilibus quae tantum uniuntur in toto per congregacionem et segregationem, et per consequens posuit continuum componi ex indivisibilibus in actu, quia indivisibilita in potentia sunt illa quae faciunt per se unum totum.” *Quaestio de continuo*, §56, 246: “[…] non componitur continuum ex indivisibilibus in actu, quia inter terminos est contradictio, continuum et indivisibile in actu, quia, si sit continuum, igitur partes eius sic si habent quod nulla est in actu per se existens separata ab alia; et si sit indivisible in actu, est per se existens separatum ab alio eiusdem rei.”

43 *Rep.* II, d. 3, q. un., 183: “Intelligo enim per esse unum esse indistinctum in se et esse distinctum a qualibet alio; et sic qualibet pars in toto est una quia distincta a qualibet alio; non dico divisum a qualibet alio, nam hoc non convenit parti in toto.”

44 *Rep.* II, d. 2, q. 3, 130: “Primum de differentia inter continua et contigua est pro me, quia dico quod si quantum non esset integratum nisi ex indivisibilibus in actu, non haberent ultimum unum sed tantum essent simul in intellecto quod Philosophus contradicit Democrito.”

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substantial parts or parts of any other kind. Thus, continuity is a physical property of really existing things and not primarily a mathematical property as it probably is in Aristotle. That continuity is a physical property of really existing things is confirmed by Chatton’s claim that it is in virtue of its nature that a continuum contains a finite number of parts, in such a way that even God cannot produce a continuum composed of an infinite number of parts.

Chatton owes his readers a redefinition of the basic notions of geometry, since his finitist atomism is not compatible with Euclidean geometry. He defines a point as an entity which does not have an intrinsic part divided in act from something having a length. Similarly, a line is an entity separated in act from any width, and a surface is an entity separated in act from any depth. In his definitions, Chatton does not make clear what he means by “separation” or “division” from the continuum. But since the normal state of a point, line, or surface is to be part of a continuum, we might speculate that Chatton is speaking of a mental separation or division, and thus is making geometrical objects fictions of the mind. In any case, Chatton’s atomism is clearly not driven by mathematical considerations on the nature of the continuum. What is important is not how two lines intersect according to Euclidean geometry but how indivisibles unite to form a whole.

Chatton seems to claim that indivisibles have zero size or zero extension; but, composed together in the right way, they form continua which do have extension. Let us examine whether this is really the case. Chatton explains that extension is the “proper accident of matter,” i.e. it is a necessary but not essential property of matter. This means that...

45 This runs counter to Murdoch’s interpretation. See for instance Murdoch, “Naissance et développement de l’atomisme,” 27: “Le plus frappant est le caractère mathématique et surtout géométrique de l’atomisme proposé et aussi bien des critiques contre cet atomisme.”

46 _Quaestio de continuo_, §82, 252: “Dico igitur quod tantum sunt finite res indivisibles in divisibles in omni re, nec Deus potest facere quod infinite res componerent aliquod continuum in mundo, nec quod in aliqua res sunt semper non tot quin plura et componere, sed componentia erunt tot quod non plura in illa re, quia potest ultra facere plura.”

47 _Rep_. II, d. 2, q. 3, 127: “punctus significat entitatem non habentem partem intrinsecam actu divisam ab habente longitudinem.”

48 _Rep_. II, d. 2, q. 3, 127: “Similiter, linea significant entitatem actu separatam ab omni latitudine, et superficies entitatem actu separatam ab omni profunditate; et quia sic non sunt separate entitates dum sunt actu partes continui, ideo continuum non componitur ex punctis etc.”

49 _Quodl_. q. 22, 147, ll. 302–3: “nihil dicitur linea, superficies vel corpus, nisi habeat alicubi partem extra partem.”

50 _Quaestio de continuo_, 259: “Quid vocas tu esse quantum? Si quod habeat partes eiusdem rationis, dico quod indivisible non habet partes, nec est quantum, quia includat contradiccionem; si quod sit pars quanti, vel quod sit talis res que cum alia re eiusdem rationis componit quantum, concedo.”

51 _Rep_. III, d. 1, q. 2, 36: “quantitas est passio materiae et coaeva sibi, et non passio totius primo.”
extension “inseparably follows matter” as heat inseparably follows fire. The extension of matter is not corruptible by natural agents.

It is difficult to determine whether Chatton’s position is restricted to the matter of natural substances or extends to prime matter considered independently of any substantial form. However, he clearly attributes a robust metaphysical independence to the extended parts of a natural substance, since he claims that they are individuated by themselves, i.e. not in virtue of a substantial form or any other kind of intrinsic individuator. This implies that, at least in theory, not only the matter of natural substances, but also prime matter, are extended by themselves, although as we saw, extension is not an essential characteristic of matter, but only something consecutive to its essence.

At first sight, Chatton’s position seems to be close to that of Averroes, who conceives of three-dimensional extension as the first and necessary determination of matter, a determination which precedes the substantial form and makes matter divisible. But extension is an accident. This raises the problem that there is an accidental property that is prior to the substance. Various solutions to this problem were given by thirteenth-century authors. Although Chatton does not address this problem explicitly, we might speculate that in

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52 Rep. IV, q. 5, a. 5, 291: “Et dico quod sic, quia [alias] non video quare corrumperetur, tum quia non minus inseparabiliter consequitur quantitas materiam quam calor ignem; sed per causas naturales non potest ignis manere sine calor suo; igitur nec materiae sine quantitate, cum aeque ipsa sit propria passio materiae sicut calor ignis.”

53 Rep. IV, q. 6, a. 1, 294: “Sed ego dico quod quantitas non potest corrumpi per accidentia naturalia, sed qualitates possunt.”

54 Rep. II, d. 5, q. un., 193: “Similiter, una parte substantiae remanente, alia potest adnihilari, igitur partes proprie substantiales se ipsis individualiter distinguantur.”

55 By contrast, when he claims that matter is extended by itself, Ockham speaks of matter informed by a substantial form. See in particular Summula, lib. 1, c. 13, ed. S. Brown, Opera Philosophica [OPh] 6 (St. Bonaventure, NY: Franciscan Institute, 1984), 192–3, ll. 39–51. Pasnau, Metaphysical Themes, 67, n. 20, rightly remarks that where the Summulae argues that matter is necessarily extended, Ockham must speak of natural necessity, because in his theological works he allows that God could deprive matter of its extension, by making all its parts exist at the same point. See Ockham, Rep. IV, q. 6, ed. R. Wood and G. Gál, OTh 7 (St. Bonaventure, NY: Franciscan Institute, 1984), 86–7. For a discussion of Ockham’s overall view on matter and extension, see Marilyn McCord Adams, William Ockham (Notre Dame, IN: Notre Dame University Press, 1987), vol. 2, 671–95; [AUTHOR].

56 Averroes, Sermo de substantia orbis, c. 1 (Venice, 1573), fols. 3vbM–4rbD: “Et quando [Aristoteles] invenit substantialias formas dividi secundum divisionem huius subiecti, divisio autem non est huic subiecto nisi in quantum habet quantitatem, scivit quod primum eorum quae existunt in hoc sunt tres dimensiones quae sunt corpus. […] Dimensiones igitur simpliciter, quae appellantur corpus simpliciter, non denudantur a prima materia.” In Metaphysical Themes, 60, Robert Pasnau reminds us that “the intuition that the extension of bodies must have its foundation in the nature of the matter of a substance inspired a variety of fourteenth-century interpretations of this Averroist view. Prime matter essentially has quantity or extension, which is here analyzed as parts outside of parts.” On this subject, see Anneliese Maier, “Das Problem der quantitas materiae in der Scholastik,” Gregorianum 27 (1946), 89–109, repr. in Maier, Die Vorläufer Galileis, 26–52; Cecilia Trifogli, “Matter and Form in Thirteenth-Century Discussions of Infinity and Continuity,” in C. Leijenhorst, C. Lüthy, and J.M.M.H. Thijsen (eds.), The Dynamics of Aristotelian Natural Philosophy from Antiquity to the Seventeenth Century (Leiden: Brill, 2002), 169–87; Silvia Donati, “The Notion of dimensiones indeterminatae in the Commentary Tradition of the Physics in the Thirteenth and the Early Fourteenth Century,” in Leijenhorst et al. (eds.), The Dynamics of Aristotelian Natural Philosophy from Antiquity to the Seventeenth Century, 189–223.
his view matter is nothing other than indivisibles that are naturally suited to unite and thus to form prime matter, given that extension is nothing other than extended things. In other words, it is in the nature of indivisibles to form continuous wholes which are prime matter and such that substantial forms emerge in it. If this is true, Chatton’s view on extension is a result of his atomism.

In any case, the crucial point of Chatton’s theory of extension is that extension does not imply impenetrability. I will explain in section two how and why Chatton believes this.

2. Extension

2.1 The Nature of Place

Suppose a natural substance that is not impenetrable. In this case, Chatton tells us, the parts of the substance would be “outside each other” (partes extra partes), i.e. spread out in space, but in such a way that nothing prevents its parts from being in the same place at the same time under the action of a natural efficient cause. Thus, a substance could have extended parts, i.e. parts outside each other, while not being solide or impenetrable.

How should extension be understood under these conditions? In a first, stronger sense, an extension is a spatial magnitude having a natural disposition to be impenetrable. In a second, weaker sense, extension is a special kind of location relation, i.e. the spatial layout of the parts relative to one another in a continuous whole. In other words, Chatton tells us, it is in the nature of an extended thing to have parts which are spatially related to each other, but it is not in the nature of an extended thing to have parts which are necessarily impenetrable to each other.

Since extension is basically a special kind of location relation, a question arises about what a location relation is. Chatton deals with this question when he examines the nature of place. Like many other fourteenth-century atomists, Chatton rejects the Aristotelian definition

57 Rep. III, d. 2, q. 1, 34: “[...] omnis res extensa est quanta omni accidente circumscripto.”
58 Rep. II, d. 3, q. un., 193: “Unde si substantia esset sine quantitate, licet bene posset una pars esse extra aliam, tamen si una moveretur per aliquid agens ad aliam per causas naturales, possent esse in eodem situ.”
59 Rep. III, d. 2, q. 1, 34: “Distinguo tamen de quanto: quia vel potest accipi pro omni illo quo posito hoc est magnum vel maius illo et extensum et habens partem extra partem, et hoc in generali sive per intrinsecam incompossibilitatem ad simul existendum cum alio in eodem situ sive non. Et in hoc sensu concedo conclusionem quod omnis res extensa est quanta omni accidente circumscripto. Alio modo pro una re quae de facto habet naturalem incompossibilitatem coexistenti cum alia. Et sic nec substantia est extensa nec qualitas eius quanta circumscripto omni alio accidente circumscripto.” See also Rep. IV, q. 4, a. 1, 275: “Concedo tamen quod substantia potest habere partem extra partem sive quantitate. Sed hoc potest dupliciter intelligi ut prius: vel scilicet quia natura sua hoc requirat, ita quod per causas naturales repugnet partes esse in eodem situ, et hoc nego, circumscripta quantitate; vel quod natura sua talem incompossibilitatem coexistenti in eodem situ non requirat, et tunc concedo.”
of place as “the innermost motionless boundary of the container.”

Instead, place, i.e. where a body is located (ubi), is the space (spatium) occupied by a body. Chatton supports his claim by arguing against Auriol’s view that place is nothing other than position in the universe (i.e. a relative thing inhering in the located body), a claim that Auriol established in answer to the well-known problem raised by Aristotle’s condition that place is immobile. First, Chatton argues, there are things, such as the heavens, which would not be located according to this theory, since the heavens are the limit of the universe and are not contained by anything. This is unacceptable. Second, even if the position of a body (as a thing distinct from the body) were removed from it, the body would still be located somewhere. In other words, it is not necessary to stipulate that position is a thing inhering in the located body in order for it to have a position. Thus, Auriol’s theory does not have any explanatory value.

In response to Auriol, Chatton claims that place is what is required for local motion, and this is nothing other than space, i.e. the space within the sides of the containing body. He explains that space is not “a space lacking bodies,” but that which would remain when the body located in it were annihilated. The possibility of a space remaining when the body located in it is annihilated presupposes that void is possible, which Chatton accepts. Indeed,


Rep. II, d. 2, q. 5, 161: “[…] spatium est per quod respondetur ad questionem factam per ubi de aliquo.”


Rep. II, d. 2, q. 5, 160: “Sed istam conclusionem non tenebo, quia non video quod sit talis passio in re locata, quia vel forset res respectiva vel absoluta; non primum, quia quid forset terminus illius respectus? Non locans vel continens, quia per te et secundum veritatem caelum non habet locans [vel] continens. Nec absoluta, quia illa destructa per potentiam Dei, adhuc res illa maneret in loco. Item, aut esset res in anima sive in imaginazione, et tunc per eam res extra animam non locatur; aut est res extra, et hoc non, quia omni re alia circumscripita posito lapide et eare continentie laps esset in loco.”

Rep. II, d. 2, q. 5, 161: “[…] illud est locus quod per se et primo requiritur ad motum locale, et illud est spatium.”

Rep. II, d. 2, q. 5, 162: “Dico quod Aristoteles vult quod locus non est spatium carens corporibus, sed non probat quin sit unum spatium quod si careret corporibus esset vacuum.”
contrary to what Aristotle believes, there can be motion in a void, defined as a “place bereft of body.” To make his case stronger, Chatton explains that an angel is not contained in a body when it is present in it, something which runs directly counter to one of the most important properties of place according to Aristotle, namely that of containment. It might be that an angel is present to a body; in any case, an angel is necessarily located somewhere because when it is not located in a body it is nevertheless in space. This does not imply, Chatton believes, that space has positive dimensions distinct from the dimensions of the located body. All this means that Chatton does not follow Aristotle’s two-dimensional view of place but develops a three-dimensional view of space. He describes space as a three-dimensional incorporeal extension, but also as an imaginary quantity, i.e. a quantity conceived in such a way that it is separated from a material body in thought.

One of the main reasons why Aristotle rejected the possibility of a vacuum conceived as “a place bereft of body,” i.e. a three-dimensional corporeal dimension, was that this hypothesis would lead to the possibility for two bodies to exist in the same place at the same time, which he deemed absurd. As we saw, Chatton avoids the assumption of an external space, i.e. a three-dimensional corporeal dimension. But, contrary to what Edward Grant suggests, the problem of interpenetration is not in this way avoided.

Co-location arguments are easily found if theological cases are advanced. Indeed, glorified bodies can move through bodies without being impeded in their motion. Chatton says that the notion of gift of subtlety (dos subtilitatis) explains why it can be so. The dos subtilitatis warrants that each part of a glorified body can correspond to a part of the body in which it is contained when it moves through it. It is not something real in the glorified

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66 Rep. II, d. 2, q. 5, 172: “Ad primum concedo quod angelus potest movere se ad libitum etiam per vacuum, si esset.” For a commentary on this text, see Robert, “Le vide, le lieu et l’espace,” 84. For this definition of vacuum, see Aristotle, Physics IV.1, 208b25.
67 Rep. II, d. 2, q. 5, 165: “[angelus] non est in loco ambiente vel continente etiamsi coexsistat corpori ambito et contento, quia non plus est angelus in aere quam e contra, quia aeque praesens est aeri sicut e contra, nec plus facit latera ambiantis distare.” For Aristotle, see Physics IV.4, 210b32–211a1.
68 Rep. II, d. 2, q. 5, 166: “licet angelus non sit in loco ambiente vel continente, tamen est in loco alter accipiendo ‘locum’, hoc est, est alicubi quia in spatio. Bene tamen concedo quod potest esse et de facto est cum corpore, sed hoc non est esse in loco.”
69 Rep. II, d. 2, q. 5, 162: “Dico quod spatium non facit latera ambientis distare, et ideo non est in loco quod hanc conditionem essendi in loco quae est hoc et huicmodi facere.”
70 Rep. II, d. 2, q. 5, 163: “Dico quod [locum] est in genere quantitatis secundum dici et secundum imaginem, quia imaginamus spatium per modum quanti.”
73 Rep. III, d. 2, q. 1, 37–8: “De corporibus autem beatorum, non pono in eis quantitatem univoce, quia illam rem non ponam ibi per quum corpus illud habeat incomposibilitatem naturalem ad simul existendum cum alio corpore, quia tunc violentaretur quando fieret cum alio corpore. Habebit igitur dotem subtilitatis loco quantitatis
bodies. The expression “gift of subtlety” is an abbreviation used in place of a sentence containing a negative term, namely the sentence “glorified bodies are deprived of the solidity (quantitas molis) that they had when they were corporeal.” If the dos subtilitatis were an accident added to bodies when they are glorified, it would not neutralize their resistance to interpenetration.74

The case of corporeal substances and accidents is not so easy to deal with. Faith leads the believer to hold that during the sacrament of the Eucharist the body of Christ and the host are in the same place at the same time and that the Eucharistic qualities are in the same place at the same time without their substance being their subject. How does Chatton explain these possibilities?

2.3 The Nature of Impenetrability

Chatton develops his answer to this question when criticizing Ockham’s physics of the Eucharist. Ockham believes that the parts of extended bodies have an internal ordering, explained in terms of an ordering of part outside part. He goes further and explains the location relation of corporeal substances in terms of relationships of containment or correspondence of parts that obtain between the located body and the space it occupies.75 Like Ockham, Chatton believes that it is in the nature of a material substance to be located in such a way that its parts are related to distinct spatial regions: there is no need to stipulate that extension is really distinct from substance or quality in order to account for the location of a substance in space, i.e. for the fact that it is extended.76 However, Chatton does not agree with Ockham, who believes that to be extended implies to be impenetrable.77 In his view, it is in

74 Quodl., q. 22. 139: “[…] quid est dos subtilitatis? Si dicatur quod est carere quantitate molis quae habetur in via, propositum, quia ex illa habet corpus incompossibilitatem essendi cum alio. Si dicatur quod sit aliqaud donum positivum, et si sic, tamen posita tali incompossibilitate essendi, consequitur quod sit incompossibile natura illi esse cum alio quidquid aliud ponatur cum illa. Sicut posita albedine in subiecto, natum est subiectum esse album, quidquid aliud ponatur. Ergo illud donum non tolleret illam incompossibilitatem essendi cum alio corpore.”

75 Ockham, Quodl. IV, q. 20, OTh 9, 400, ll. 46–9: “[…] substantia materialis ex hoc ipso quod est immediate praesens quantitatis sic quod tota est praesens toti et pars parti est immediate praesens loco illius quantitatis per partes suas intrinsecas.” See also Ockham, Tractatus de corpore Christi [TCC], c. 16, ed. C. Grassi, OTh 10 (St. Bonaventure, NY: Franciscan Institute, 1986), 122–4.

76 Quaestio de continuo, 238: “[…] quantitas nihil est nisi partes extense.”

virtue of an accident really distinct from a substance and its qualities, namely quantity or solidity, that the substance located in such a way coincides part by part with the spatial region it occupies.78

Chatton advances several arguments in favor of his view, which are all directed against Ockham’s view that solidity is no more distinct from material substance or quality than extension. Let us begin by spelling out in more detail the relation between extension and solidity according to Ockham. Concerning extension, Ockham states that “every extended material substance is composed of substantial parts distant from one another in place or location.”79 A material substance has extended parts by its nature and not in virtue of quantity as an accident inhering in it. Wood, for instance, has parts by its nature and not in virtue of something inhering in wood. Moreover, when something quantified (such as a substance or a quality) is in a place, it is circumspectively in a place, that is, in such a way that the whole quantified thing is in the whole place and each part of it is in a part of place.80 This implies that it is in the nature of a body not to spatially overlap with another body, i.e. it is in the nature of a body not to be in exactly the same place circumspectively as another body; no further accident in the category of quantity has to be stipulated in order to account for this. The solidity of a body is thus nothing more than the fact that its parts cannot spatially overlap due to their spatial distribution in the place occupied by the body.

In order to defend his reductionist claim, Ockham proposes a thought experiment based on the theological claim that God can preserve a substance without any accident inhering in it and without moving that substance by local motion.81 Suppose that God preserves all the accidents of one half of a piece of wood and removes all the accidents of the other half. On this hypothesis, Ockham claims, the half deprived of its accidents would not undergo locomotion. Otherwise, it would penetrate the other half or separate from it, which runs counter to the initial stipulation that the substance does not undergo locomotion. Consequently, a material substance is extended and solid by itself, and there is no need to

78 Rep. III, d. 2, q. 1, 36: “Item, arguitur sic: substantia extensa est in loco circumspective per partes suas intrinsecas etc. – Dico quod si per esse in loco circumspective intelligas habere naturalem incompossibilitatem ad simul existendum in eodem situ, nego. Bene tamen per naturam suam, hoc est hic et illud ibi, et non per additum, licet non incompossibiliter etc.” See also Quodl. q. 22, 146, ll. 247–50: “Dicendum quod non ponitur quantitas ut sit substantia praesens loco, sed ad salvandum quod per causas naturales, substantia non potest esse simul localiter cum corpore locante ipsam.”
79 Ockham, TCC, c. 12, OTh 10, 112, ll. 6–7. For a commentary on this text, see Adams, Some Later Medieval Theories of the Eucharist, 153–62; Pasnau, Metaphysical Themes, 288–93; [AUTHOR].
80 TCC, c. 16, OTh 10, 123–4, ll. 50–60.
81 Ockham, Tractatus de quantitate, q. 3, OTh 10, 54–6; TCC, c. 26, OTh 10, 146–9. Chatton reports this argument in Quodl. q. 22, 142–3, ll. 131–56.
stipulate an accident in the category of quantity in order to account for this. By the principle of parsimony, one must conclude that quantity (understood as the property of being both extended and solid) is not a thing distinct from substance.\(^{82}\)

Ockham also believes that qualities are quantified by themselves, and not by means of an accident of quantity that would make both the substance and its other accidents quantified. Ockham claims that during the sacrament of the Eucharist the Eucharistic qualities (or species) must be intrinsically extended. Indeed, the host is as impenetrable as before its consecration, although they are without a subject and the quantity of the substance has disappeared with the substance during transubstantiation, since it was not really distinct from it.\(^{83}\)

Chatton wants to prove that one can grant the result of Ockham’s thought experiment without granting that the Eucharistic species are intrinsically solid. Chatton’s line of thought is distinctive among Ockham’s realist opponents. For instance, the Pseudo-Campsall and Walter Burley try to show that Ockham’s thought experiment does not give the expected result.\(^{84}\) Chatton grants it, as is expected from his view on extension, but does not grant the conclusion that Ockham derives from it.\(^{85}\)

He begins by reporting Ockham’s basic line of thought in favor of his claim: the quantity of the substance of the Eucharistic species has disappeared with the substance during transubstantiation; since the Eucharistic species are impenetrable during the sacrament, their impenetrability does not come from their substance; therefore, not only the substance but also its qualities are intrinsically extended.\(^{86}\) Chatton then counteracts Ockham’s razor with his anti-razor: he intends to prove that what Ockham posits in his ontology is not sufficient to account for the facts, and consequently that there are some things other than substances and qualities that are necessary, namely accidents in the category of quantity. Chatton’s claim is

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\(^{82}\) Ockham, *Tractatus de quantitate*, q. 3, OTh 10, 58 and *TCC*, c. 29, OTh 10, 157–8, ll. 9–16.

\(^{83}\) Ockham, *TCC*, c. 24, OTh 10, 143, ll. 5–15.


\(^{85}\) Rep. III, d. 2, q. 1, 35: “[…] concedo, sicut et Doctor Subtilis, quod Deus possit facere substantiam sine omni accidente absoluto, et ultra concedo quod potest conservare partes substantiae tunc in distinctis sitibus, quia teneo quod aliae sunt partes substantiae et aliae quantitatis.”

\(^{86}\) Rep. III, d. 2, q. 1, 35: “Sed contra ista arguitur primo sic: nihil est quantum nisi sit quantitas vel subjectum quantitatis vel forma recepta in quantitate. Sed qualitas in Eucharistia est quaedam forma extensa, quia fides tenet quod accidentia ibi sunt sine subiecto, nec quantitas recipitur in eis; igitur quaelibet illarum qualitatum est una quantitas.”
the following: the proposition “Two Eucharistic species are impenetrable during the Sacrament” cannot be made true by two qualities alone; consequently, given that their substance has disappeared due to transubstantiation, an accident of the category of quantity which accounts for their impenetrability must be posited.\(^87\)

Chatton substantiates his claim with several arguments. I will follow the order chosen by Ockham in his report of them in his *Quodlibet V*, q. 26, because it systematizes them in a very convenient way.\(^88\) First, after the consecration the parts of the Eucharistic species are naturally impenetrable; but this impenetrability does not come from the species as qualities. Indeed, qualities of different species, such as whiteness and coldness, are not impenetrable in the host, given that they are in the same place at the same time. Moreover, qualities can become more or less intense, i.e. they can acquire new parts or lose parts, while remaining in the same place. There must be something really distinct from the qualities of the host that explains why some qualities in the host are not impenetrable while others are. Thus, qualities are not impenetrable by themselves but by something added to them, namely an accident of quantity.\(^89\)

Second, Ockham cannot account for the fact that in the host all the qualities are in the same place at the same time. Let us posit a whiteness in a host and let us suppose that God posits in the same place another host without any quality inhering in it. Is this host able to remain in the same place as the whiteness or not? If this is the case, the host had this property before its consecration, which does not correspond to the facts, given that in Ockham’s account a quality cannot be in the same place at the same time with a substance other than the one in which the quality inhere(d), according to the principle that an accident does not migrate from one subject to another. If this is not the case, it is necessary to stipulate something really


\(^{88}\) Ockham, *Quodl.* IV, q. 26, OTh 9, 425–7.

\(^{89}\) *Lectura* I, d. 8, q. 1, 25: “Assumptum patet, nam illae quantitates non sunt substantiae extensae, quia post consecrationem non sint ibi substantiae extensae nec qualitates, quia qualitates illarum hostiarum non sunt se ipsis incompossibiles naturaliter ad essendum simul, sicut patet, quia albedo et frigiditas hostiae sunt simul secundum situm, quae sunt qualitates alterius rationis; etiam partes intensivae albedinis sunt qualitates eiusdem rationis, et tamen sunt simul secundum situm; igitur istae qualitates non sunt quantitates; quia quantitates sunt se ipsis incompossibiles ad essendum simul in eodem situ naturaliter, sive sunt eiusdem rationis sive alterius, ideo philosophantes habuerunt pro magno inconvenienti quod quaecumque dimensiones penetrarent se.” See also *Quodl.* q. 22, 140, ll. 63–71.
distinct from it in order to account for it. This thing must be a quantity, i.e. that which makes something impenetrable.  

Third, the Eucharistic qualities have the same physical behavior after consecration as before; in particular, new qualities can be produced and the existing species can be intensified. But Ockham admits that qualities cannot be naturally active if they do not have a subject. This is why he accepts that it is God who miraculously maintains the physical powers of the qualities in the consecrated host. But, Chatton objects, if we follow this explanation, all certitude that is obtained by means of sensation would perish, which would give believers an occasion of doubting their faith. Therefore, it is better to follow the common view of the theologians and accept that quantity is the immediate subject of the Eucharistic species. This way, the qualities of the host can be said to have their natural power during the sacrament and there is no need to appeal to God’s miraculous intervention. 

In a fourth and last argument, Chatton generalizes his criticism from extended qualities to substances. The Bible tells us that glorified bodies go through the heavens. This implies that it is possible for a body—such as glorified bodies—to be in the same place as another body, namely the heavens. But if a corporeal substance were impenetrable by itself, then the bodies of the blessed could not move or they would move violently when they go through the heavens. 

90 Rep. III, d. 2, q. 1, 31: “Contra: si albedo in Eucharistia sit una talis res quae habet naturalem incompossibilitatem coexistendi cum aliis corporibus, volo quod Deus ponat unam aliam substantiam panis sine omnibus talibus accidentibus, et quaero an illa habeat naturalem incompossibilitatem ut per causas naturales fiat in eodem situ cum accidentibus illis separate, aut non. Si sic, igitur et substantia panis quae praefuit, habuit naturalem incompossibilitatem existendi cum eis, quia incompossibilitas illa est passio specifica, et ideo si convenit uni, cuilibet alteri individuo eiusdem speciei saltem aeque intenso potest competere simile secundum regulam eorum. Si non, igitur ad hoc quod ista albedo sit naturaliter incompossibilis ad essendum cum illa substantia requiritur alia res et nulla qualitas sufficient nec substantia est ibi; igitur per illam rem quam voco quantitatem; et hoc est propositum.”

91 Rep. IV, q. 5, a. 1, 285: “Hic dicetur quod ad praesentiam ignis bene producitur ibi aliquis gradus caloris, sed Deus est ibi totale activum, non ignis. Sed nolo hoc dicere, quia ista responsio daret infidelis occasionem discrendi plus quam oporteret dare ex virtute sacramenti. Quicumque enim dicet quem albedo illa non ageret in visum nec videretur, esset ministrare maiorem occasionem infidelis discrendi quam requirit sacramentum; sed tantam occasionem ministraret dicere quod illa hostia non posset alterari nec pati, quia ex hoc sequeretur aeque quod species vini non posset frigefacere linguam meam, quia si sic, et omnes agens naturale in agendo repatitur, igitur etc. Sed hoc negare esset dicere quod non sentiretur naturaliter ibi frigus, quod est aequale inconveniens cum primo; igitur etc.” See also Quodl. q. 22, 141, ll. 99–107.


93 Quodl., q. 22, 139: “Alia est opinio vera: quod praeter quamcumque substantiam et qualitates est unum accidens ponendum quod est causa incompossibilitatis naturalis duorum corporum simul in eodem situ. […] Antecedens patet de corporibus glorificatis etc. Si dicatur quod inter duo corpora glorificata est naturalis incompossibilitas essendi simul sic est hic, tamen Deus facit talia corpora esse simul. Contra: natura corporis citius expelleret alium quam esset simul cum eo, ergo non minus est disconveniens naturae eorum quod sint simul
Most of Chatton’s arguments are based on the possibility of co-located extended entities, i.e. entities existing in the same place at the same time. Co-location arguments of this kind will have force against anyone who, like Ockham, wants to explain the impossibility of co-location merely in terms of extension. So if the arguments are accepted, the result is that impenetrability must be distinguished from extended substances and qualities.

This is precisely Chatton’s conclusion. To have what Chatton calls extension is simply to have parts that are located at a distance from one another. This is intrinsic to both material substances and qualities. There is, however, something else which Chatton also calls quantity: having parts that are not just spread out but that resist co-location. Thus, Chatton does postulate real accidents in the category of quantity, although he also accepts the fundamental metaphysical thesis of Ockham’s reductive project, namely that extension is a basic feature of material entities.

But is Chatton’s position really tenable? One could object that given his views on extension he should not grant Ockham’s thought experiment. For what would be lost once quantity is removed from one half of the wood is the power of the parts of the wood to resist co-location. They might stay where they are, spread out in the normal non-co-located way, but there would no longer be anything keeping them from drifting inward on top of each other if a body were to penetrate them by local motion. I do not believe that this is a satisfactory result: in a sense, Chatton’s view on extension makes of bodies physical entities that behave according to the laws of the mechanics of fluids rather than according to Newton’s physics.
Moreover, these objections do not seem to reach their target, given that Ockham has straightforward answers to them. In response to the first objection, Ockham makes his view on the impenetrability of qualities more explicit. Only parts of quality spread out in different parts of their subject are impenetrable; when they are deprived of their subject, there is no reason why they would not remain as impenetrable to each other as before. In response to the second objection, Ockham says that qualities are not impenetrable with the substance in which they inhere, and this comes from their nature: there is no other explanation to be sought, because the principle that an accident does not migrate from one subject to another is a basic assumption. The third objection, Ockham says, applies as much to Chatton as to himself. Indeed, Chatton’s account of condensation and rarefaction presupposes that one cannot assign a subject to the change of extension, since according to him extension is not really distinct from material substance. So, during the sacrament of the Eucharist, if the extension of the host condenses or rarefies, this is because of God’s miraculous assistance.

Ockham’s answer to the last objection is particularly interesting, because it seems to confirm my view that Chatton’s distinction between extension and solidity is not well grounded. Ockham begins by distinguishing two ways in which something can be solid or impenetrable—something that he would not have done if Chatton had not criticized him. In the first way, a thing is impenetrable if it has a natural active power to move in order not to be penetrated by another body. In the second way, a thing is impenetrable if it is by its nature not able to be penetrated by another body without any external action. In other words, in the second sense, when a thing is penetrated by another body under the action of an external

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96 Ockham, *Quodl*. IV, q. 26, OTh 9, 430, ll. 128–34: “Ad secundum principale dico quod partes qualitatis iam productae in diversis partibus subjecti distantibus situ sunt ita naturaliter incompossibiles in eodem situ sicut partes quantitatis mediae; et ideo si illae partes separentur a subiecto et conserventur, sicut est in Eucharistia, non plus possent per naturam fieri in eodem situ quam possunt quando fuerunt in diversis partibus subiecti.”

97 Ockham, *Quodl*. IV, q. 26, OTh 9, 430, ll. 141–8: “Ad tertium dico quod illae species post consecrationem sunt naturaliter incompossibiles cum alia substantia in eodem situ, sicut patet ad sensum quod alia substantia eis cedit. Et hoc forte est quia non sunt naturaliter compossibiles cum aliqua substantia in eodem situ nisi cum illa in qua primo producuntur sicut in subiecto. Sed tunc non sequitur: alia substantia est incompossibilis in eodem situ; igitur substantia in qua praefuerunt est eis incompossibilis in eodem situ.”

98 Ockham, *Quodl*. IV, q. 26, OTh 9, 431–2, ll. 166–75: “Ad quartum dico quod idem argumentum est contra te, ponendo quantitatem de rarefactione et condensatione quantitatis ubi non potes dare subiectum. Ideo pro omnibus talibus potest dici quod omnia apparentia sensui quae fiunt circa hostiam non-consecratam, per ordinationem divinam fiunt circa hostiam consecratam immediate a Deo, ex quo non possunt fieri a potentia creatae. Dico igitur quod tam augmentatio quam productio novae qualitatis fiunt totaliter et immediate a Deo, sicut tu ponis de rarefactione.”

99 Ockham, *Quodl*. IV, q. 26, OTh 9, 432, ll. 186–94: “Ad quartum dico quod unum corpus substantiale habere naturalem incompossibilitatem existendi cum alia substantia potest dupliciter intelligi: uno modo, quia si existeret in eodem situ cum alio corpora, habet virtutem activam naturalem movendi se ne existat cum illo corpora, sicut grave movendi [se] sursum; et si sic haberet substantia naturalem incompossibilitatem existendi cum alio corpora, tunc violenter esset cum alio corpora. Sed sic nulla substantia habet naturalem incompossibilitatem existendi cum alio.”
cause, this does not happen violently. This explains why glorified bodies can penetrate other corporeal substances without any violent change.\textsuperscript{100}

This means that Ockham seems to grant Chatton’s objection: there must be a weak sense of “impenetrability” that includes the possibility for a glorified body to move through the heavens without any violent change. But Ockham does not grant his conclusion, namely that quantity must be a thing really distinct from substance or quality. Put another way, in order to account for the possibility of the motion of the glorified bodies through heaven, it is not necessary to stipulate that they lack a thing really distinct from them, namely quantity. We just have to arrange our definition of impenetrability so that the possibility of the special case of the glorified bodies is not ruled out by definition.

Is Ockham justified in making the physics of glorified bodies an exception to the standard physical behavior of bodies? In his view, the term “body” can in some circumstances pick out things (namely, glorified bodies) that are able to go through other bodies, i.e. bodies that are more “flexible” than the standard objects of mechanics. When the glorified bodies move through the heavens, God acts miraculously to make them penetrable. Ockham has a good reason to think so: giving too much weight to the case of the glorified bodies would amount to accepting too high a price for the physics of solids. As a consequence, Ockham affirms that Chatton’s view is not true: his objections are not conclusive, which implies that Chatton is not right in using his anti-razor against his view on quantity.\textsuperscript{101}

2.4 Condensation and Rarefaction

Given his view on the solidity of bodies, one would expect Chatton to give a distinctive explanation of the phenomena of condensation and rarefaction. I believe, however, that his explanation is not satisfactory. Let me explain why. In medieval natural philosophy, a body is said to condense when it becomes less extended without losing any material, and to rarefy when it becomes more extended without gaining new material.\textsuperscript{102} If the accident of quantity were replaced by another during condensation and rarefaction, then given that quantity is the immediate subject of qualities, all qualities would change as soon as a body condenses or

\textsuperscript{100} Ockham, \textit{Quodl.} IV, q. 26, OTh 9, 432–3, ll. 195–202: “Aliter dicitur habere incompossibilitatem naturalem, quia existens per se non potest naturaliter se facere in eodem situ cum alio corpora, nec etiam existens in uno situ cum alio corpora potest naturaliter se facere in loco alio per se. Et quod sic habet incompossibilitatem naturalem, existens in eodem situ cum alio corpora, non est ibi violenter; et sic una substantia habet incompossibilitatem naturali existendi cum alia in eodem situ, et sic est de corporibus gloriois.”

\textsuperscript{101} Ockham, \textit{Quodl.} IV, q. 26, OTh 9, 427, ll. 59–62: “Sed ista opinio non videtur vera, quia quando propositio verificatur pro rebus etc.; sed partes substantiae essentiales et qualitates salvant omnia apparentia quae per fidem salvari possunt. Nec istaes rationes concludent.”

\textsuperscript{102} On condensation and rarefaction, see Maier, \textit{Die Vorläufer Galileis}, 26–52; Donati, “The Notion of \textit{dimensiones indeterminatae},” 141–69.
rarefies. How, then, are condensation and rarefaction to be accounted for? Chatton explains that during this change the accident of quantity in the condensed or rarefied substance remains constant and that only its extension changes. In other words, there is no correlation between a change in impenetrability and a change in extension. The main reason is that condensation and rarefaction are cases neither of substantial change nor of quantitative augmentation.

How, then, is the change of extension to be accounted for? For both Ockham and Chatton, the substance changes its extension because it coexists with a smaller or greater place than before the change. In Ockham’s explanation, the parts of the substance are more or less distant than before. Given that for Ockham the parts of a body have by themselves a fixed volume, they cannot just be squeezed. This implies that the parts overlap if the substance is condensed, which is unacceptable according to Chatton. This is why, Chatton believes, during this change the substance gains or loses extended parts (corpora). This is

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105 Rep. IV, q. 5, a. 4, 293: “Ad probationem dico quod stant simul cum identitate quantitatis quod nunc partes plus distant, nun minus, propter maiorem quod in imbibitionem minorem corporum.”

106 Rep. IV, q. 5, a. 2, 287: “Item, quare non maneret eadem? Subiectum manet idem, scilicet materia; nec habet contrarium per quod possit corrupti; nec forma requirit hoc, quia forma ita bene rarefit et coexsistit maiori loco sicut materia. Similiter, rarefactio est motus distinctus ab augmentatione tam substantiali quam quantitativa; sed hoc nonnisi quantitas maneret eadem.”

107 Rep. IV, q. 5, a. 2, 287: “Ad rationes opinionis contrariae respondendum est per distinctionem de quantitate pro re distincta et pro coexsistere maiori vel minori loco, sive magni vel parvo loco etc.”


109 Rep. IV, q. 5, a. 3, 288: “Alle est opinio quod partes non plus distant quam prius in rarefactione nec propinquius iacent in condensatione, quia non possunt propinquius iacere quam quod partes sint unum vel simul.” For Ockham, the overlapping of the parts in condensation is the result of a local motion of the parts. See Ockham, Summula, lib. 1, c. 13, OP, n. 6, 194. For an analysis of Ockham’s view on condensation and rarefaction that is analogous to Chatton’s interpretation, see [AUTHOR]. For a different interpretation of Ockham’s view, see Pasnau, Metaphysical Themes, 303.

110 Rep. IV, q. 5, a. 3, 288: “Verumtamen quem non esset obligatus nisi ad naturalem rationem correspondentem sensui et non ad auctoritatem diceret quod numquam fit condensatio nisi per egressionem corporum de poris rei. Petat enim ad sensum quod totum corpus condensatum est modo in mediate et spatio quo prius et iam est in situ in quo prius pars eius; igitur vel partes sunt simul vel una adhî nihilatur, quorum utrumque est inconveniens, vel aliquod corpus egreditur loco cuius recipiuntur partes prius distantes, et hoc est propositum. Item, partes circumferentialia propinquius iacent quam prius, et non itur in infinitum, si opinio de indivisibilius sit vera;
not to say that the extension of the substance becomes more or less gappy. In his explanation there is no need for an inter-particular void, as maintained by Nicholas of Autrecourt for instance. Nicholas explains condensation by the local movement of atoms inside the compound, which causes the separation of parts already present in the body and the concomitant introduction of a void between these atoms.\footnote{This description of Nicholas of Autrecourt’s position comes from Christoph Grellard, “Nicolas of Autrécourt’s Atomistic Physics,” in C. Grellard and A. Robert (eds.), Atomism in Late Medieval Philosophy and Theology (Leiden: Brill, 2009), 124–6.} In Chatton’s view, the same substance occupies more or less space because it gains or loses matter.

If this interpretation is correct, a problem arises from Chatton’s atomist claim that any change in the amount of matter entails a change in identity. Indeed, according to what we saw in section one, it seems that having such-and-such an amount of matter is necessary for the identity of a continuum, such that a variation in amount entails a change in numerical identity. Thus, condensation and rarefaction raises a special problem for Chatton’s metaphysics of continuity, but Chatton seems not to be aware of it.

If Chatton truly holds that we can regard a continuum as numerically one thing, and if its numerical unity depends on the nature of the parts of the continuum, then there is also a biological counterexample to his view on quantity, namely the phenomenon of nutrition, usually described as “an augmentation of the substance.” Nutrition involves a change from food to part of a living body. At the end of the process of nutrition, the new flesh becomes a part of the preexisting flesh. Is this process a generation or a mere change of volume of the preexisting substance?

Chatton must find some principled way of distinguishing nutrition from mere increase of quantity by means of rarefaction. He claims that nutrition involves the replacement of one substantial form by another, which implies that the former accident of quantity of the changed part of the body is corrupted and that a new accident of quantity is generated. What is added is thus “a new substance having its own quantity.”\footnote{Lectura I, d. 17, q. 6, 386: “Ideo dicendum quod in augmentatione substantiae compositae ex materia et forma acquiritur aliquid novum, quia nova materia aliminti ponitur iuxta materiam membrorum, et de illa materia noviter unita educitur nova forma substantialis conveniens membro. Ibi enim est nova quantitas, scilicet [quae] concomitantur materiam illam noviter unitam. Patet ergo quod hic addatur nova substantia habens suam quantitatem.”} So, a change in material (or more precisely, organic) composition is sufficient for the existence of a new substance. When he has finished eating his dinner, Socrates is not the same (sum of) substance(s) as before.
This has to be confirmed, however, by an investigation of Chatton’s view on identity over time, in particular his answer to the question whether there are any substances whose homogeneous parts are necessary for their identity. This seems to be obvious for homogeneous substances such as water, but it remains unclear for organic or living substances. In any case, Chatton grants the point: there is a true substantial change at the end of the process of nutrition, a change which is a necessary condition for any true change of the accident of quantity.  

Conclusion

Chatton’s finitist atomism is driven by his belief that anyone who holds that the continuum is infinitely divisible also accepts the existence of created infinites, which he firmly rejects. His position is also motivated by his anti-Democritean stance: impenetrability is not reducible to mere atomistic structure, and atoms compose a continuum not by means of mere random local motion. Chatton uses his finitist atomism to explain concrete physical processes, which confirms that it is not primarily motivated by a mathematical analysis of continuous quantity.

The metaphysical picture resulting from these choices is the following. Prime matter is constituted by unextended indivisibles. Indivisibles, united in the appropriate way, form continuous wholes. In this metaphysical picture, extension has a special function. It emerges on the natural joining of indivisibles in the appropriate way. It is in virtue of extension that a thing’s parts are spread out in a continuous and unified way; that is, it is what makes a continuous whole extended. Moreover, extension, according to Chatton, entails an ordering of parts in a whole, such that each part exists “outside the others.”

Chatton’s view on extension is all the more interesting in that it is based on a distinction between extension and impenetrability. The distinction highlights that in the conceptual analysis of impenetrability two conceptions of impenetrability must be distinguished, namely the impenetrability that results from the internal structure of the body, and the impenetrability that must be stipulated in order to account for local motion. In the end, even if Ockham does not accept the conclusion that Chatton draws from this distinction—namely, that impenetrability is a thing really distinct from substance or quality—he does

113 Lectura I, d. 17, q. 6, 378: “Secundo, hoc probatur de augmentacione substantiae, qui cum alimentum convertitur in substantiam rei alendae, nihil ibi acquiritur de novo, quia si sic, maxime hoc videtur de quantitate, sed quantitas non, quia ipsa non acquiritur sine generatione substantiae, quia non adventit materiae dum informatur forma praecedente; ergo forma praecedens corrupitur, et per consequens nova inducitur, ergo quantitas non est nova sine nova generatione substantiae.”
acknowledge that this very distinction has a real explanatory function. From this perspective, the dialogue between Ockham and Chatton shows us that reductionist views on quantity are more diverse than Ockham’s realist opponents want us to believe.