A model of calorimetric measurements in an open quantum system

Paolo Muratore-Ginanneschi

Department of Mathematics and Statistics University of Helsinki

In collaboration with

Brecht Donvil, Antti Kupiainen, Jukka Pekola, Kay Schwieger.

CNR Roma, October 29, 2018
Outline

1. Calorimetric measurement on a driven qubit
2. Mathematical modeling
3. Results
4. Outlook
Calorimetric principle

Idea: measure work statistics in an Open Quantum System\textsuperscript{a}

\textsuperscript{a}Pekola et al., “Calorimetric measurement of work in a quantum system”, 2013.

- protocols bringing the system back to the initial state at the end of the horizon.
- the work $W$ done on the system under these conditions is equal to the heat $Q$ dissipated to the environment
Calorimetric measurement on a driven qubit

Stylized experimental setup

- Qubit
- Drive
- Calorimeter
- Phonon bath

\[ \hbar \omega_q \]
\[ H_d(t) \]
\[ H_I \]
\[ H_{ep} \]
\[ T_e \]
\[ T_p \]
Integrated quantum circuit


---

Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Qubit driven by a monochromatic force

\[ H_q(t) = \frac{\hbar \omega_q}{2} \sigma_z + \kappa V_d(t) \]
\[ V_d(t) = \hbar \omega_q (e^{i \omega_L t} \sigma_+ + e^{-i \omega_L t} \sigma_-) \]

\[ \kappa \hbar \omega_q = \text{drive amplitude} \quad \omega_L = \text{drive frequency} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Calorimeter: free fermion gas (effectively, more follows)

\[ H_e = \sum_k \eta_k c_k^\dagger c_k \]
\[ \eta_k = \frac{\hbar \| k \|^2}{2m} \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

Qubit calorimeter interaction

\[ H_{qe} = g \frac{\sqrt{8 \pi} \epsilon_F}{3N} \sum_{k \neq l \in S} (\sigma_+ + \sigma_-) c_k^\dagger c_l, \]

\[ N = O(10^9) \text{ fermions} \]

\[ S = \text{energy shell around } \epsilon_F \]
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

**Phonons**

\[ H_p = \sum_k \hbar \omega_k b_k^\dagger b_k \]

\[ \omega_k = v_s k \]

\( v_s = \text{sound speed} \quad k = \|k\| \text{ phonon wavelength norm.} \)
Closed system description

\[ H = H_q + H_e + H_{qe} + H_p + H_{ep} \]

(Herbert) Fröhlich’s Hamiltonian

\[ H_{ep} = \lambda \sum_{k,q} \omega_q^{1/2} \left( c_k^\dagger c_{k-q} b_q + c_k^\dagger c_{k-q} b_q^\dagger \right) \]
Mathematical modeling

Timescales

- \( \tau_{ee} = O(10^0) \text{ns} \): Landau quasi-particle relaxation rate to Fermi–Dirac equilibrium in a metallic wire.
- \( \tau_{ep} = O(10^4) \text{ns} \): electron-phonon interactions.
- \( \tau_R = 2 - 5 \times O(10^5) \text{ns} \): transmon qubit relaxation times (Wang et al., *Applied Physics Letters*, (2015))
- \( \tau_{eq} \sim g^{-2} \)
  Fermi’s golden rule estimate of characteristic qubit-calorimeter time scale.

Open quantum system approach

\[ \tau_{ee} \ll \tau_{eq} \ll \tau_{ep} \ll \tau_R \]
Phonon–fermion bath interaction

- Phonon bath temperature \( T_p = O(10^{-1}) \) K (cryostat)
- Fermion bath temperature \( T_e \)

\[
T_e \simeq T_p
\]

**Mean energy current** \( \propto T_p^5 - T_e^5 \) (leading order\(^a\))

**Rms energy current fluctuations** \( \propto O(T_p^3) \) at \( T_e = T_p \) (leading order\(^b\))


Idea of the model

**Qubit:** stochastic Schrödinger equation\(^a\)


\[ d\psi = (\text{deterministic dissipative drift}) \, dt + \text{Poisson jumps} \]

**Calorimeter:** equilibrium Fermi–Dirac ensembles at evolving \(T_e^a\)


\[ dT_e^2 = \frac{1}{N \gamma} dE \]  

Sommerfeld expansion

\[ dE = dE_{eq} + dE_{ep} = \text{Poisson jumps} + (T_p^5 - T_e^5) dt + O(T_p^3) dw_t \]
Stochastic Jump Process

Closed system: unitary evolution

\[ \psi(t + dt) - \psi(t) = d\psi(t) = -iH\psi dt \]

Open quantum system: stochastic Schrödinger equation\(^{234}\)

- Fermi golden rule: **dissipative terms** are added to the Hamiltonian

\[ H\psi(t)dt \rightarrow G(\psi(t))dt \]

- Fermi golden rule: transitions induce **stochastic jumps**

\[ (|\pm\rangle - \psi(t))dN(\mp\omega), \quad dN(\mp\omega) = 0, 1, \]

\[ E_\psi(dN(\mp\omega)) = \gamma(\mp\omega)||A(\mp\omega)\psi||^2dt \]

Weak-drive approach: add drive as a perturbation to the continuous evolution

\[ G(\psi(t)) + \kappa H_d(t)\psi(t) \]


Temperature Process

Using the Sommerfeld expansion we find the dependence of the temperature on the change in internal energy $E$ of the calorimeter

$$dT_e^2 = \frac{dE_{eq} + dE_{ep}}{\gamma}.$$ 

The qubit-electron interaction alone ($dE_{ep} = 0$) gives

$$dE_{eq} = \hbar \omega (dN(\omega) - dN(-\omega)),$$

$$d\psi(t) = -i[G(\psi(t)) + \kappa H_d(t)\psi(t)]dt$$

$$+ \left( |+\rangle - \psi(t) \right) dN(-\omega) + \left( |-\rangle - \psi(t) \right) dN(\omega)$$
Upshot of the modeling

"Strong drive": Floquet theory


- $\tau_{qe} \gg \tau_m = \text{inverse separation of peaks in the radiation spectrum (RWA)}.$
- Resonant drive: $\tau_m/\tau_{qe} \sim g^2/\kappa \ll 1$
- Temperature+population process: jump diffusion master equation

"Weak drive"

- $g^2/\kappa \geq 1$
- Temperature+state process: hybrid master equation

Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1K$

Temperature distributions after 10 periods of resonant strong drive
Short-time temperature behaviour
Initial temperature of the electron bath: \( T_e = 0.1 K \)

(Left) Mean temperature of the calorimeter after 10 periods of driving vs driving frequency \( \omega_L \) for different values of the qubit calorimeter coupling \( g \). Stars= weak-drive. Lines: Floquet . (Right) Standard deviation.
Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1K$

Comparison of temperature distributions after 10 periods.

P. Muratore-Ginanneschi (Helsinki Univ.)
model of calorimetric measurements
Helsinki 2018 13 / 20
Relaxation to a steady state

The qubit-calorimeter reaches a steady state.
Effective temperature process

**Multiscale expansion:** \( \varepsilon \propto \frac{1}{N} \) & \( s = \varepsilon t \geq O(1) \)

\[
dT_e^2 = \frac{1}{\gamma} \left( \Sigma V(T_p^5 - T_e^5) + J(T_e^2) \right) ds + \frac{1}{\gamma \sqrt{N}} \left( \sqrt{10\Sigma V k_B T_p^3} + \sqrt{S(T_e^2)} \right) dw_s
\]

**Analytic estimates**

\[
\langle T_e \rangle \approx \left( T_p^5 + g^2 \frac{O(\hbar \omega_L^2)}{\Sigma V} \right)^{1/5}
\]

mean steady state temperature

\[
\tau \approx \left( T_p^5 + g^2 \frac{O(\hbar \omega_L^2)}{\Sigma V} \right)^{-3/5}
\]

relaxation time to steady state
Numerics vs analytic theory (Floquet)

Steady state temperature PDF

$\sigma = 0.004 \, \text{K}$

$\xi = 0.001 \, \text{K}$

$g^2 = 0.005$

$\sigma = 0.005 \, \text{K}$

$\xi = 0.003 \, \text{K}$

$g^2 = 0.05$
Steady state av. temperature vs drive strength

\[ T_s (K) \]

\[ g^2 = 0.1 \]

\[ g^2 = 0.01 \]

\[ g^2 = 0.001 \]

Floquet

Weak drive
Outlook

- Predictions always involve weak coupling between qubit and calorimeter.
- Perturbative Markovian master equation techniques not reliable beyond the strictly weak subsystem-bath coupling limit (see e.g. Segal, *Physical Review B*, (2013)).
- Strong qubit-calorimeter coupling analysis desirable.
References

Based on

- B. Donvil, P. Muratore-Ginanneschi, J. P. Pekola *Hybrid master equation for calorimetric measurements* 2018

THANKS FOR YOUR ATTENTION
THANKS, Brecht, Antti, Jukka & Kay