A model of calorimetric measurements in an open quantum system

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Outline

1. Calorimetric measurement on a driven qubit
2. Mathematical modeling
3. Results
4. Outlook

Calorimetric principle

Idea: measure work statistics in an Open Quantum System[a]

[a] Pekola et al., "Calorimetric measurement of work in a quantum system", 2013.

- protocols bringing the system back to the initial state at the end of the horizon.
- the work W done on the system under these conditions is equal to the heat Q dissipated to the environment

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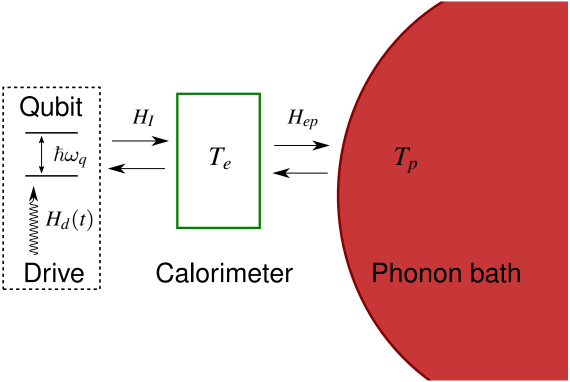
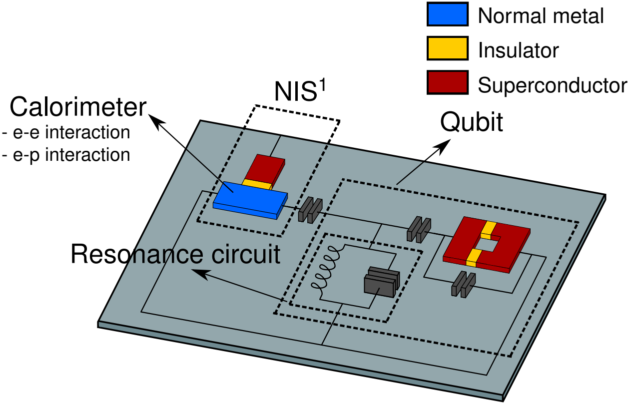
Stylized experimental setup



Integrated quantum circuit



Envisaged experimental implementation (Pekola et al., *New Journal of Physics*, (2013), Gasparinetti et al., *Physical Review Applied*, (2015), Viisanen et al., *New Journal of Physics*, (2015)

[1] Schmidt, Schoelkopf, and Cleland, "Photon-Mediated Thermal Relaxation of Electrons in Nanostructures", 2004.

Closed system description

$$H = H_q + H_e + H_{qe} + H_p + H_{ep}$$

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Qubit driven by a monochromatic force

$$H_q(t) = \frac{\hbar\,\omega_q}{2}\sigma_z + \kappa\, V_d(t)$$
$$V_d(t) = \hbar\omega_q\left(e^{\imath\omega_L t}\sigma_+ + e^{-\imath\omega_L t}\sigma_-\right)$$

$\kappa\,\hbar\,\omega_q =$ drive amplitude

$\omega_L =$ drive frequency

Closed system description

$$H = H_q + H_e + H_{qe} + H_p + H_{ep}$$

Calorimeter: free fermion gas (effectively, more follows)

$$H_e = \sum_k \eta_k \, c_k^\dagger \, c_k$$

$$\eta_k = \frac{\hbar \|\boldsymbol{k}\|^2}{2\,m}$$

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Closed system description

$$H = H_q + H_e + H_{qe} + H_p + H_{ep}$$

Qubit calorimeter interaction

$$H_{qe} = g\,\frac{\sqrt{8\,\pi}\,\epsilon_F}{3\,N}\sum_{k \neq l \in \mathbb{S}} (\sigma_+ + \sigma_-)\,c_k^\dagger\,c_l,$$

$N = O(10^9)$ fermions

$\mathbb{S} =$ energy shell around ϵ_F

Closed system description

$$H = H_q + H_e + H_{qe} + H_p + H_{ep}$$

Phonons

$$H_p = \sum_k \hbar\,\omega_k\, b_k^\dagger\, b_k$$

$$\omega_k = v_s\, k$$

$v_s =$ sound speed

$k = \|\boldsymbol{k}\|$ phonon wavelength norm.

Closed system description

$$H = H_q + H_e + H_{qe} + H_p + H_{ep}$$

(Herbert) Fröhlich's Hamiltonian

$$H_{ep} = \lambda \sum_{\boldsymbol{k},\boldsymbol{q}} \omega_q^{1/2} \left(c_{\boldsymbol{k}}^{\dagger}\, c_{\boldsymbol{k}-\boldsymbol{q}}\, b_{\boldsymbol{q}} + c_{\boldsymbol{k}}^{\dagger}\, c_{\boldsymbol{k}-\boldsymbol{q}}\, b_{\boldsymbol{q}}^{\dagger} \right)$$

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Timescales

- $\tau_{ee} = O(10^0)\text{ns}$:
  Landau quasi-particle relaxation rate to Fermi–Dirac equilibrium in a metallic wire.
- $\tau_{ep} = O(10^4)\text{ns}$:
  electron-phonon interactions.
- $\tau_R = 2 - 5 \times O(10^5)\text{ns}$: transmon qubit relaxation times (Wang et al., *Applied Physics Letters*, (2015))
- $\tau_{eq} \simeq g^{-2}$
  Fermi's golden rule estimate of characteristic qubit-calorimeter time scale.

Open quantum system approach

$$\tau_{ee} \lll \tau_{eq} \lll \tau_{ep} \lll \tau_R$$

Phonon–fermion bath interaction

- Phonon bath temperature $T_p = O(10^{-1})\mathrm{K}$ (cryostat)
- Fermion bath temperature T_e

$$T_e \simeq T_p$$

mean energy current $\propto T_p^5 - T_e^5$ (leading order[a])

rms energy current fluctuations $\propto O(T_p^3)$ at $T_e = T_p$(leading order[b])

[a] Kaganov, Lifshitz, and Tanatarov, “Relaxation between Electrons and the Crystalline Lattice”, 1957; Wellstood, Urbina, and Clarke, “Hot-electron effects in metals”, 1994.

[b] Pekola and Karimi, “Quantum noise of electron-phonon heat current”, 2018.

Idea of the model

Qubit: stochastic Schrödinger equation[a]

[a]Breuer and Petruccione, *The Theory of Open Quantum Systems*, 2002.

$$\mathrm{d}\psi = (\text{deterministic dissipative drift})\,\mathrm{d}t + \text{Poisson jumps}$$

Calorimeter: equilibrium Fermi–Dirac ensembles at evolving T_e[a]

[a]Berg, Brange, and Samuelsson, "Energy and temperature fluctuations in the single electron box", 2015; Marinari and Parisi, "Simulated Tempering: A New Monte Carlo Scheme", 1992.

$$\mathrm{d}T_e^2 = \frac{1}{N\,\gamma}\mathrm{d}E \qquad \text{Sommerfeld expansion}$$

$$\mathrm{d}E = \mathrm{d}E_{eq} + \mathrm{d}E_{ep} = \text{Poisson jumps} + (T_p^5 - T_e^5)\mathrm{d}t + O(T_p^3)\mathrm{d}w_t$$

Stochastic Jump Process

Closed system: unitary evolution

$$\psi(t+\mathrm{d}t)-\psi(t)=\mathrm{d}\psi(t)=-iH\psi\mathrm{d}t$$

Open quantum system: stochastic Schrödinger equation[2,3,4]

- Fermi golden rule: **dissipative terms** are added to the Hamiltonian

$$H\psi(t)\mathrm{d}t\to G(\psi(t))\mathrm{d}t$$

- Fermi golden rule: transitions induce **stochastic jumps**

$$(|\pm\rangle-\psi(t))\,\mathrm{d}N(\mp\omega),\quad \mathrm{d}N(\mp\omega)=0,\ 1,$$
$$\mathrm{E}_{\psi}(\mathrm{d}N(\mp\omega))=\gamma(\mp\omega)\|A(\mp\omega)\psi\|^{2}\mathrm{d}t$$

Weak-drive approach: add drive as a perturbation to the continuous evolution

$$G(\psi(t))+\kappa H_{d}(t)\psi(t)$$

[2] Ghirardi, Rimini, and Weber, “Unified dynamics for microscopic and macroscopic systems”, 1986.

[3] Dalibard, Castin, and Mölmer, “Wave-function approach to dissipative processes in quantum optics”, 1992.

[4] Breuer and Petruccione, *The Theory of Open Quantum Systems*, 2002.

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Temperature Process

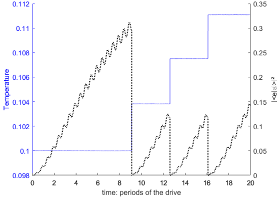
Using the Sommerfeld expansion we find the dependence of the temperature on the change in internal energy E of the calorimeter

$$\mathrm{d}T_e^2 = \frac{\mathrm{d}E_{eq} + \mathrm{d}E_{ep}}{\gamma}.$$

The qubit-electron interaction **alone** ($\mathrm{d}E_{ep} = 0$) gives

$$\mathrm{d}E_{eq} = \hbar\omega(\mathrm{d}N(\omega) - \mathrm{d}N(-\omega)),$$

$$\begin{aligned} \mathrm{d}\psi(t) = & - i[G(\psi(t)) + \kappa H_d(t)\psi(t)]\mathrm{d}t \\ & + \Big(|+\rangle - \psi(t)\Big)\mathrm{d}N(-\omega) + \Big(|-\rangle - \psi(t)\Big)\mathrm{d}N(\omega) \end{aligned}$$



Upshot of the modeling

"Strong drive": Floquet theory[a]

[a]Breuer and Petruccione, “Dissipative quantum systems in strong laser fields: Stochastic wave-function method and Floquet theory”, 1997.

- $\tau_{qe} \gg \tau_m$ =inverse separation of peaks in the radiation spectrum (RWA).
- Resonant drive: $\tau_m/\tau_{qe} \simeq g^2/\kappa \ll 1$
- Temperature+population process: jump diffusion master equation

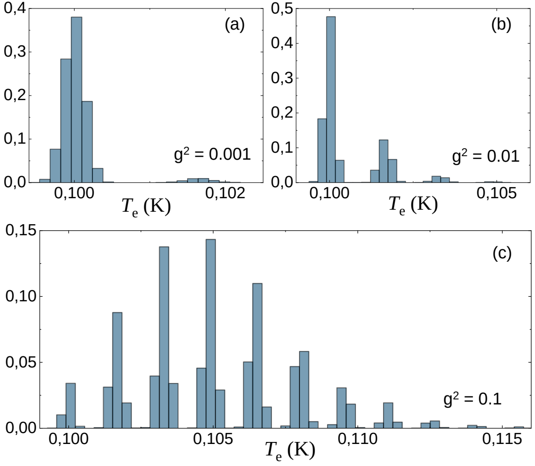
"Weak drive"

- $g^2/\kappa \geq 1$
- Temperature+state process: hybrid master equation[a]

[a]Chruściński et al., “Dynamics of Interacting Classical and Quantum Systems”, 2011.

Short-time temperature behaviour

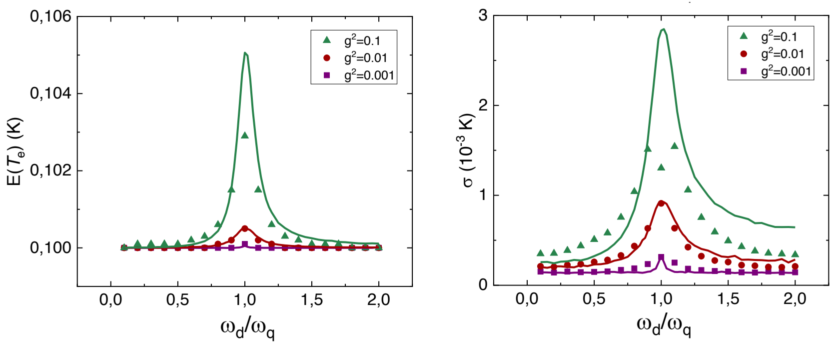
Initial temperature of the electron bath: $T_e = 0.1K$



Temperature distributions after 10 periods of resonant strong drive

Short-time temperature behaviour

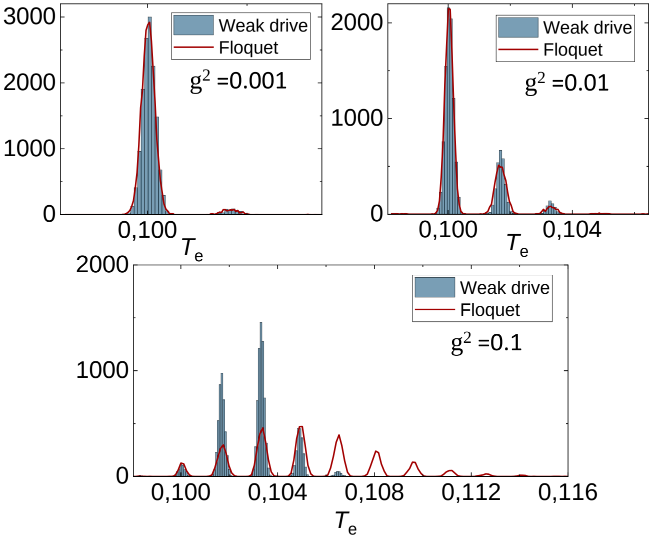
Initial temperature of the electron bath: $T_e = 0.1K$



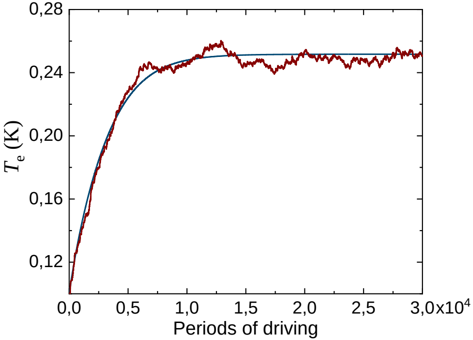
(Left) Mean temperature of the calorimeter after 10 periods of driving vs driving frequency ω_L for different values of the qubit calorimeter coupling g. Stars= weak-drive. Lines: Floquet . (Right) Standard deviation.

Short-time temperature behaviour

Initial temperature of the electron bath: $T_e = 0.1K$



Relaxation to a steady state



The qubit-calorimeter reaches a steady state.

Effective temperature process

Multiscale expansion: $\varepsilon \propto 1/N$ & $s = \varepsilon\, t \geq O(1)$

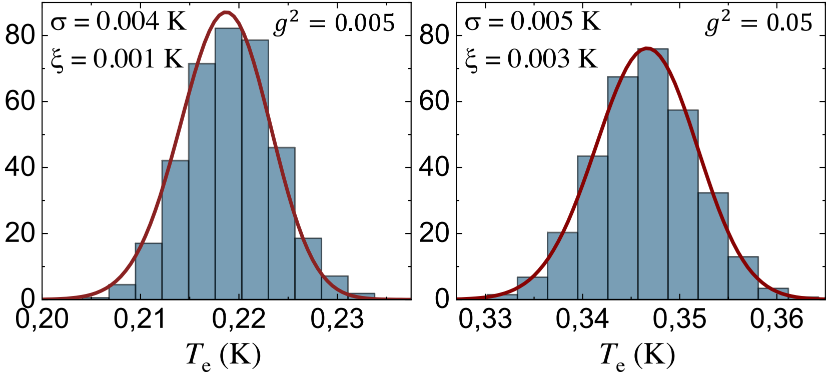
$$\mathrm{d}T_e^2 = \frac{1}{\gamma}\left(\Sigma V(T_p^5 - T_e^5) + J(T_e^2)\right)\mathrm{d}s + \frac{1}{\gamma\sqrt{N}}\left(\sqrt{10\Sigma V k_B}T_p^3 + \sqrt{S(T_e^2)}\right)\mathrm{d}w_s$$

Analytic estimates

$$\langle T_e \rangle \approx \left(T_p^5 + g^2\frac{O(\hbar\omega_L^2)}{\Sigma V}\right)^{1/5}$$ mean steady state temperature

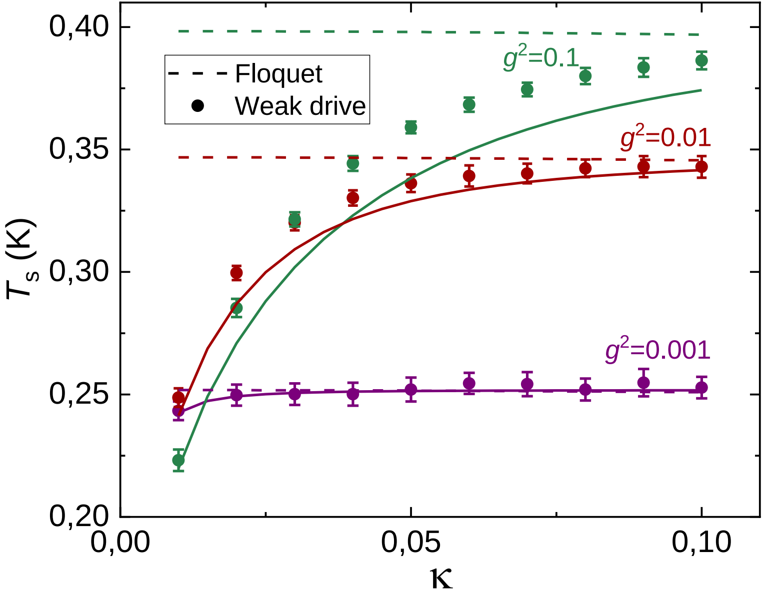
$$\tau \approx \left(T_p^5 + g^2\frac{O(\hbar\omega_L^2)}{\Sigma V}\right)^{-3/5}$$ relaxation time to steady state

Numerics vs analytic theory (Floquet)



Steady state temperature PDF

Steady state av. temperature vs drive strength



Outlook

- Predictions always involve weak coupling between qubit and calorimeter.
- Perturbative Markovian master equation techniques not reliable beyond the strictly weak subsystem-bath coupling limit
(see e.g. Segal, *Physical Review B*, (2013)).
- Strong qubit-calorimeter coupling analysis desirable.

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References

Based on

- A. Kupiainen, P. Muratore-Ginanneschi, J.P. Pekola, and K.Schwieger *Fluctuation Relation for Qubit-Calorimetry* arXiv:1606.02984, and Phys. Rev. E. 94, 062127, (2016).
- B. Donvil, P. Muratore-Ginanneschi, J. P. Pekola, and K. Schwieger, *A model for calorimetric measurements in an open quantum system* arXiv:1803.11015 and Phys. Rev. A 97, 052107 (2018).
- B. Donvil, P. Muratore-Ginanneschi, J. P. Pekola *Hybrid master equation for calorimetric measurements* 2018

THANKS FOR YOUR ATTENTION

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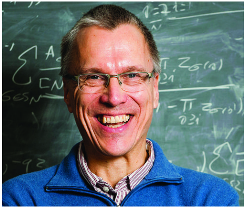
THANKS, Brecht, Antti, Jukka & Kay



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