Teaching and Teacher Education 79 (2019) 137–152

Contents lists available at ScienceDirect

Teaching and Teacher Education

journal homepage: www.elsevier.com/locate/tate

Using network analysis methods to investigate how future teachers conceptualize the links between the domains of teacher knowledge

Mika Koponen [a, *], Mervi A. Asikainen [b], Antti Viholainen [b], Pekka E. Hirvonen [b]

[a] University of Helsinki, Faculty of Educational Sciences, P.O.Box 9, FI-00014, Helsinki, Finland
[b] University of Eastern Finland, Department of Physics and Mathematics, P.O.Box 111, FI-80101, Joensuu, Finland

HIGHLIGHTS

- A new approach to investigating teacher knowledge is presented.
- Mathematical Knowledge for Teaching (MKT) is used for classifying knowledge issues.
- Network analysis is used for examining relationships between teacher knowledge.
- The six MKT domains exist in a hierarchical sequence in the minds of future teachers.
- The approach would also be applicable in other subjects.

ARTICLE INFO

Article history:
Received 25 August 2016
Received in revised form
28 November 2018
Accepted 10 December 2018

Keywords:
Teacher knowledge
Mathematics teacher education
Mathematics education
Mathematical knowledge for teaching
Network analysis

ABSTRACT

In this article we present a new approach to investigating teacher knowledge. The essay data related to Finnish future teachers' (N = 18) perceptions of the "knowledge required for teaching mathematics" were transformed into a network. We classified the knowledge topics using the Mathematical Knowledge for Teaching (MKT) framework and examined the relationships between the issues raised with the aid of network analysis. According to the results, the future teachers see the six MKT domains in a hierarchical sequence. As it is not subject specific, this approach is also applicable in the investigation of teacher knowledge of other subjects.

1. Introduction

One of the theoretical perspectives from which the contents of teacher education and the knowledge possessed by teachers can be examined is *teacher knowledge* (see Neubrand, 2018). The course contents of teacher education have an impact on teachers' knowledge, teachers' knowledge has an impact on teaching, and teaching has an impact on students' learning and achievements in mathematics (e.g. Hill, Rowan, & Ball, 2005; Fung et al., 2017; Schmidt, Houang, & Cogan, 2011; Tchoshanov et al., 2017). The results of the first large-scale[1] international study of teacher knowledge, TEDS-M (*Teacher Education and Development Study in Mathematics*), also indicate that it has an effect on students' achievements in mathematics. TIMSS results are noticeably better in countries in which teachers' knowledge is stronger (Schmidt, Houang, et al., 2011).

In the domain of mathematics, several frameworks have been developed to describe the knowledge required for teaching this subject (e.g. Ball, Thames, & Phelps, 2008; Baumert & Kunter, 2013; Ernest, 1989; Fennema & Franke, 1992; O'Meara, 2011; Rowland, Turner, Thwaites, & Huckstep, 2009). Researchers have agreed that teachers need strong subject knowledge, and they all share the view that mathematics teachers also need a different kind of knowledge from that required by mathematicians. They consider that pedagogical knowledge is needed in the teaching of mathematics. However, in the domain of mathematics, teacher

* Corresponding author.
E-mail addresses: mika.koponen@helsinki.fi (M. Koponen), mervi.asikainen@uef.fi (M.A. Asikainen), antti.viholainen@uef.fi (A. Viholainen), pekka.e.hirvonen@uef.fi (P.E. Hirvonen).

[1] Approximately 5000 teacher educators and 22 000 future teachers from 750 programs in some 500 teacher education institutions in 17 countries participated in the TEDS-M study (e.g. Tatto et al., 2008).

https://doi.org/10.1016/j.tate.2018.12.010
0742-051X/

knowledge frameworks reveal similarities with Shulman's ideas, and hence several frameworks can be understood to be elaborating on, rather than replacing, Shulman's (1986) conceptualizations (Petrou & Goulding, 2011). Although the existing teacher knowledge frameworks describe the knowledge required for effective teaching quite well, less attention has been paid to how the categories of teacher knowledge are interconnected, in other words, the actual structure of teacher knowledge (e.g. Baumert & Kunter, 2013; Hashweh, 2005).

According to Fennema and Franke (1992), teacher knowledge is a dynamic entity, the categories of which actively influence each other. Ball et al. (2008) have also noted that many of the demands imposed by teaching mathematics require knowledge of the intersection of the six knowledge types of the framework of *Mathematical Knowledge for Teaching*. For example, recognizing students' incorrect answers falls under Common Content Knowledge (CCK), identifying the nature of an error comes under Specialized Content Knowledge (SCK), and knowledge of typical errors is an aspect of Knowledge of Content and Students (KCS) (Ball et al., 2008). Mishra and Koehler (2006) have suggested that teachers' knowledge of subject, pedagogy, and technology are independent knowledge types, but also that a mixture of these knowledge types constitutes a type of knowledge in its own right. On the basis of these perspectives, we can claim that the categories of teacher knowledge may be interrelated in a variety of ways, but that there is no consensus regarding the way in which the various categories of teacher knowledge are interconnected.

Baumert and Kunter (2013) have argued that, since the interconnections of teacher knowledge are unclear, the structure of teacher knowledge is also only vaguely known. Hence, the problem of which aspects of teacher knowledge should be learned before new topics can be understood remains unsolved. Nevertheless, according to the framework provided by the *Ladder of Knowledge*, the first step is subject matter knowledge, the second is pedagogical knowledge, and the last is knowledge of effective teaching (O'Meara, 2011). O'Meara (2011) develops a strong argument in response to the question of how such knowledge is structured: "*…it does not make good sense… to make teachers believe that they can make a full scale assault on pedagogical content knowledge without first acquiring a strong content knowledge*" (p. 192). Similarly, in the *Knowledge Quartet,* mathematical knowledge and beliefs refer to *Foundation Knowledge*, while each of the other three domains is based on Foundation Knowledge (Rowland et al., 2009). Familiarity with the relevant Foundation Knowledge is a prerequisite for teachers intending to teach mathematics (Rowland et al., 2009). It makes sense that knowledge is somewhat hierarchical and learning new knowledge requires prior knowledge, but investigating the interconnections and structural features of teacher knowledge in detail may very well also require fresh approaches for the conduct of the research itself.

We believe that we have developed a new approach to investigating teacher knowledge and its interconnections by using network analysis methods. Our approach interprets teacher knowledge as a network in which *nodes* indicate the knowledge required for teaching and *arrows* indicate how the knowledge components interconnect with each other. This approach appears to be new, and even unique, within the wider research field focused on teacher knowledge, since respondents' beliefs about the knowledge required for teaching (nodes) and their perceptions concerning the ways in which knowledge topics are interconnected (arrows) can both be included in the same data, and simultaneously analyzed. In addition, nodes and arrows form various storylines in the network, and these storylines can explain not only *what kind* of knowledge is needed for teaching, but also *why* and *in which context* such knowledge is needed.

In the present study, we focus on investigating final-year future teachers' perceptions of the kind of knowledge required for teaching mathematics. Investigating this particular group has proved interesting since the TEDS-M study compared future teachers' perceptions of the nature of the learning opportunities that their teacher education offered them to the kind of knowledge that future teachers hold. Interestingly, future teachers' perceptions of such learning opportunities were related to the knowledge that they hold (Schmidt, Cogan, & Houang, 2011; Schmidt, Houang, et al., 2011). Hence, it is reasonable to assume that the contents of teacher education impact not only on future teachers' knowledge but also on their perceptions of the knowledge required for teaching. Although future teachers' perceptions of the knowledge needed for effective teaching may not predict how these teachers will teach in the future, an investigation of their perceptions may help reveal potential development needs in the contents of teacher education. It has been observed in Finland that novice mathematics teachers may find that, when viewed from the perspective of their actual education, their mathematical and pedagogical knowledge are not very well interconnected. As a result, they report experiencing difficulties in applying their learned knowledge in actual teaching situations (Koponen, Asikainen, Viholainen, & Hirvonen, 2016). Transforming theoretical knowledge into the practice of teaching is a universal challenge in teacher education, an observation made by several studies (e.g. Korthagen, 2010; Korthagen & Kessels, 1999).

In this article, we present a new approach to investigating future teachers' perceptions of the knowledge required for teaching and its interconnections. Using this approach, we have been able to detect a number of new and unexpected structural features of teacher knowledge in the realm of mathematics. As the approach is not subject specific, it may also be applicable in investigating teacher knowledge in other subjects.

2. Theoretical framework

Mathematical Knowledge for Teaching (MKT) is a practice-based theory of mathematics teacher knowledge (e.g. Ball et al., 2008; Hill, Ball, & Schilling, 2008). According to MKT, teachers need six different kinds of knowledge for teaching mathematics (Fig. 1).

Common Content Knowledge (CCK). Teachers need a broad, competent knowledge of mathematics, that is, mathematical theories, concepts, terms, definitions, rules and symbols. Furthermore, they should be able to derive concepts, calculate and prove theorems. All of these aspects of subject knowledge are important for teachers, as they would also be for other related occupations, for example, mathematicians. Hence, these aspects are termed Common Content Knowledge (Ball et al., 2008).

Specialized Content Knowledge (SCK). Teachers commonly use a variety of mathematical representations, and visualize mathematics and create connections between the ways in which mathematics can be represented. If teachers have a historical knowledge of mathematics or if they know how mathematics can be applied, they are able to use this information in their teaching (Jankvist, Mosvold, Fauskanger, & Jakobsen, 2015; O'Meara, 2011). Furthermore, teachers need to select relevant examples and appropriate exercises for every situation and to evaluate how these work in practice. All these aspects require mathematical knowledge of a kind that is unique to teaching, and are generally termed Specialized Content Knowledge (Ball et al., 2008).

Horizon Content Knowledge (HCK). Teachers require knowledge of the structure of mathematics, such as how concepts are hierarchically related and how together, these concepts form topics. However, teachers also need to be aware of how mathematics is constructed for their students, for example, which concepts

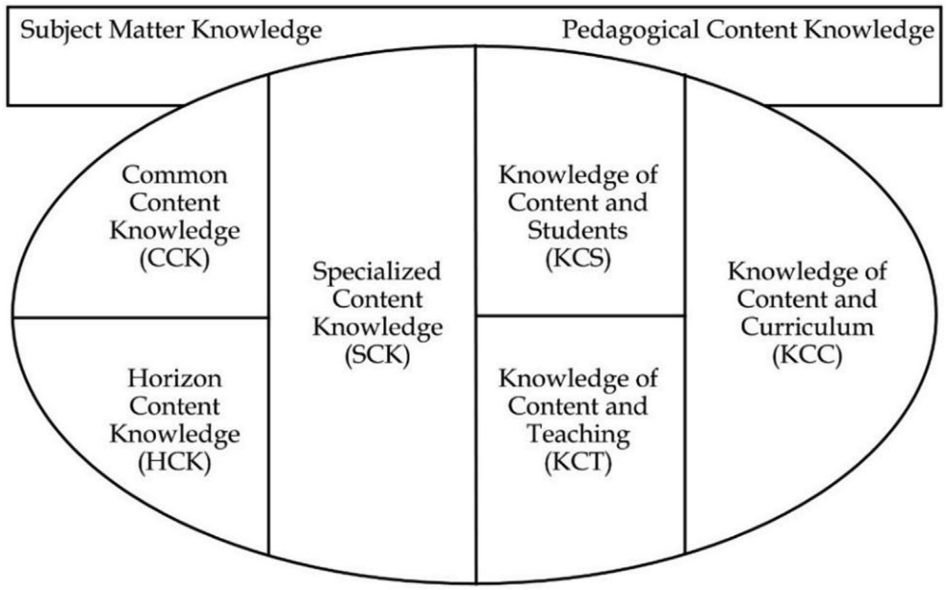


Fig. 1. Domains of mathematical knowledge for teaching (MKT).

students already know and which concepts they will learn later. In other words, teachers need to know what prior knowledge is needed for the student to learn new areas of mathematics. These aspects are related to the structure of both mathematics and the learning of mathematics, and are hence regarded as aspects of Horizon Content Knowledge (Ball & Bass, 2009; Ball et al., 2008).

Knowledge of Content and Students (KCS). Because teachers need to gain an understanding of how their students learn mathematics in theory, they must also have some knowledge of learning theories. On the other hand, teachers need to recognize whether students are liable to face challenges in learning mathematics or whether they face particular challenges in their learning, such as learning difficulties. Teachers need to know their students, understand their approaches to learning, and recognize the various kinds of challenges that they face. In sum, teachers need to know some of the ways in which they can motivate their students and promote their learning. These aspects require an understanding of how students think, know or learn particular content. This aspect of teacher knowledge is referred to as Knowledge of Content and Students (Ball et al., 2008; Hill et al., 2008).

Knowledge of Content and Teaching (KCT). Teachers need to know and choose teaching methods for each situation. Hence, they need to have knowledge of planning lessons, communicating and promoting interaction in the classroom. Teachers also need to be able to change their teaching strategy, organize special learning support for students and improve their own teaching. All these aspects are related to planning and organizing the kind of teaching that requires an amalgam of knowledge of teaching and mathematics, and hence this knowledge is referred to as Knowledge of Content and Teaching (Ball et al., 2008).

Knowledge of Content and Curriculum (KCC). The national curriculum normally supplies guidelines for specific teaching, and thus teachers should be familiar with the contents of their national curriculum. However, teachers should also have knowledge and skills relevant to the use of teaching materials (such as textbooks, other materials, etc.), teaching instruments (blackboards, overhead projectors, etc.), and technology (computers, smart boards, calculators, software, etc.). All these aspects of knowledge can be summed up in terms of Knowledge of Content and Curriculum (Ball et al., 2008; Jankvist et al., 2015).

The MKT framework is based on Shulman's conceptualization (1986), and its foundations lie within an American context. Today, however, the use of MKT as a framework for teacher knowledge has also been distributed to other countries, e.g. Ireland (Delaney, Ball, Hill, Schilling, & Zopf, 2008), South Korea (Kwon, Thames, & Pang, 2012), Ghana (Cole, 2012), Indonesia (Ng, 2012), Norway (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012), Iceland (Jóhannsdóttir & Gísladóttir, 2014), Finland (Koponen et al., 2016), and Malawi (Kazima, Jacobsen, & Kasoka, 2016). Although MKT is quite a popular framework for describing teachers' knowledge, several questions still require further attention: (a) What kind of teaching tasks require SCK? (Ball et al., 2008; Orrill et al., 2015); (b) What is the relationship between CCK and SCK for teachers at different school-levels? (Dreher, Lindmeier, & Heinze, 2016; Dreher, Lindmeier, Heinze, & Niemand, 2018; Speer, King, & Howell, 2015); and (c) What are the exact definitions of the MKT domains? (Ball et al., 2008; Markworth, Goodwin, & Glisson, 2009).

We selected the MKT framework as the contextual theory for this study because it has already been used in a similar way to classify teacher knowledge (Markworth et al., 2009). It has also been used in the Finnish context for classifying teacher educators' and mathematics teachers' perceptions of the knowledge required for teaching mathematics (Koponen et al., 2016) and for assessing teacher educators' and mathematics teachers' perceptions of the kind of knowledge teachers have learned in the course of their education (Koponen, Asikainen, Viholainen, & Hirvonen, 2017).

In the present study, the future teachers' essays on *the kind of knowledge needed for teaching mathematics* were first converted into a network, and then the relationships between the issues raised were examined using network analysis. The aim was to find answers to two research questions:

1. According to the views expressed by future teachers, what kind of knowledge is needed for teaching mathematics?
2. How are knowledge domains related to each other in the minds of future teachers?

3. Method

3.1. Context and data collection

The University of Eastern Finland offers two different kinds of programs for future mathematics teachers. The subject teacher program provides the conventional qualification of a mathematics subject teacher. The hybrid mathematics teacher program provides a dual qualification for prospective mathematics subject teachers as well as primary school teachers. Hence, students in the hybrid mathematics teacher program take more pedagogical studies than those aiming to become conventional mathematics teachers. Both these teacher education programs offer masters-level degrees that are required for recognized teachers to work in Finland.

The data for this study were collected during a course called *Analysis skills for teaching mathematics*, which was aimed towards both future subject teachers and future hybrid teachers. The contents of the course offered elementary knowledge of graph theory suitable for senior high school students combined with an analysis of the teaching of graph theory. The course instructor taught graph theory for 2 h after which the future teachers were required to analyze his teaching for a following 2 h. This cycle was repeated eight times. The idea is similar to that presented in Morris' (2006) study, but instead of analyzing videotaped mathematics lessons, the future teachers analyzed face-to-face teaching. The future teachers were given no direct instructions on how to implement their analysis but they were asked to build their own model that worked for analyzing and developing the teaching of mathematics. The course served as a context for learning to analyze the teaching of mathematics. Graph theory was selected because this mathematics topic is not taught in Finnish schools. Since graph theory was a new topic for future teachers, they would experience the actual learning of mathematics in a more credible manner, thus enabling them to analyze the teaching of graph theory in finer detail.

At the outset of the course, the future teachers wrote essays under the rubric of *The kind of knowledge needed for teaching mathematics*. These essays were then returned to them and they were asked to share their thoughts and discuss the essay topics in small groups. The future teachers were also requested to make notes on these conversations, and the course instructor encouraged them to “steal” others' ideas, but only if the stolen idea matched their own thinking. These 2-h small group sessions were repeated three times during the course, each time in small mixed groups, which meant that each future teacher had an opportunity to talk to everybody at least once. At the end of course, the future teachers were asked to reflect on their ideas, this time individually, by writing essays about the kind of knowledge needed for teaching. In the context of Finnish teacher education, it is normal during courses for future teachers to interact and discuss their findings with each other. In particular, at the University of Eastern Finland the number of students participating in the mathematics teacher education programs is small (the average annual number of mathematics subject teacher graduates is nineteen) and students usually know each other quite well. In consequence, the future teachers always have a mutual influence on each other. As we could not avoid this peer influence, we ensured that every future teacher had the opportunity to discuss the topics with their peers at least once during the course. It is, in consequence, very possible that the future teachers learned something new about teacher knowledge during the course that they took, but in each situation, the course instructor highlighted that future teachers must reflect their own thinking and approve all new ideas. In the present study, we look in depth at the final essays of the future teachers.

A total of 18 future teachers participated in the study. All the participants gave consent for their essays to be used for the purposes of our research. They were Finnish, their average age was 24, and half of the participants were male and half female. Eight were future subject teachers and 10 future hybrid teachers (see Table 1).

However, as Table 1 indicates, both student groups were at a rather similar stage when they participated in the course. They had taken almost two years of mathematics studies at university level and had completed almost all of the compulsory mathematical studies (e.g. calculus, analysis, algebra, differential equations). They had studied teaching and learning in general but had also focused on the didactics of mathematics and had completed over half of their pedagogical studies and teaching practice. Some of the future teachers in both groups had also worked as subject teachers at school level. Since both of the future teacher groups were at a relatively similar stage in their studies, in this article we ignore the small differences between the two groups.

3.2. Analysis

3.2.1. Transforming text data into the form of a network

Text data can be transformed into a network in many ways. For example, network evaluation, map analysis, knowledge graphing and functional depiction can all be used for this purpose (Popping & Roberts, 1997). The first two approaches are better suited to quantitative research, whereas the latter two are more appropriate for qualitative research (Popping & Roberts, 1997). In *Knowledge graphing,* textual data are transformed into a network in which the nodes describe knowledge topics and the links between the nodes represent how these topics are interconnected (Popping, 2003). For knowledge graphing, text data can be analyzed either with or without the aid of computers. For example, in *Linguistic Inquiry and Word Count,* a computer is used to count words, compare written words with dictionaries, analyze word contexts and detect meaningful categories for words (Tausczik & Pennebaker, 2010). In contrast, *Latent Semantic Analysis* is a fully automatic text data analysis method that transforms each word and its text passage into a matrix, and then with the aid of factor analysis, examines the relationships between the words and their contexts (Landauer, Foltz, & Laham, 1998). Although the Google algorithm has shown that text data can be analyzed remarkably well, not all text analysis algorithms achieve the same level of effectiveness. Hence, an alternative approach to investigating large quantities of text data and to classifying the text within an adequate number of categories representing similar meanings is that of qualitative content analysis (Hsieh & Shannon, 2005). As the Finnish language is challenging for many text analysis algorithms, in practice we preferred to use qualitative content analysis for our knowledge graphing.

First, we read the essay data several times to achieve immersion and to obtain a sense of the whole (Tesch, 1990). Thereafter, were identified all the knowledge topics mentioned in the essays and code them. Repeated topics – for example, many future teachers mentioned that teachers needed to “know mathematical theories” – were classified within the same data-based category. This kind of data-based analysis is known as *Conventional Content Analysis* (see Hsieh & Shannon, 2005). Next, the data-based issues were classified on the basis of the various domains of MKT. This theory-based classification is identified as *Direct Content Analysis* (Hsieh & Shannon, 2005). The described analysis process is similar to that reported in a study by Markworth et al. (2009), in which the interview responses and conversational topics were coded on the basis of the domains of MKT. Another study by the Koponen et al. (2016) similarly reports how the teachers' and their educators' answers to open-ended questions were classified according to the domains of MKT. After this, the essay data were read again. This time, we tracked how the knowledge topics identified were related

Table 1
Background information on future teacher participants (N = 18).

	N	Stage of studies in years	Completed mathematical studies	Completed pedagogical studies	Completed teaching practice	Complete studies altogether	Do you have experience of working as a school teacher?
Future subject teachers	8	90% (4.5 of 5 years)	85% (103 of 120 cp)	60% (36 of 60 cp)	67% (2.75of 4 stages)	78% (234 of 300 cp)	Yes[a] for 38% (3/8)
Future hybrid teachers	10	80% (4.0 of 5 years)	78% (93 of 120 cp)	64% (77 of 120 cp)	60% (2.4 of 4 stages)	84% (252 of 300 cp)	Yes[a] for 50% (5/10)

[a] Refers to temporary posts and total work experience, ranging from a few hours to a month.

to each other (examples are shown in Table 2). The analyses yielded a total of 136 knowledge issues (nodes) and 364 relations (arrows).

Some researchers have found that the definitions and boundaries of MKT categories are sometimes unclear (see Koponen et al., 2016; Markworth et al., 2009). Ball et al. (2008, p. 403) also claim "*It is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions*". As there is a risk that data-based categories do not match the domain definitions of MKT, we will present all data-based categories and their classification into the MKT domains in the results section.

Next, we converted the data into Gephi format. Gephi software is designed for exploring and manipulating large networks, and its format is similar to the non-standardized *Comma-Separated Values* (.csv) file type, in which a comma separates text or values into tabular cells (Bastian, Heymann, & Jacomy, 2009; Khokhar, 2015). In our case, we use a comma to, for example, distinguish the section of an essay, produce classifications into data-based and theory-based categories and to provide details about the respondents.

3.2.2. Network analysis using Gephi software

Importing the data into the Gephi software resulted in one large network, consisting of 136 nodes and 364 arrows between these nodes. The *nodes* were the knowledge issues mentioned by the future teachers in their essays, whereas the *arrows* marked the ways in which the knowledge issues were connected to each other (see final column in Table 2).

To answer the first research question, we focused on the ways in which the different parts of the network were connected to each other. In the context of network analysis, strongly connected parts of the network are referred to as *communities* (Blondel, Guillaume, Lambiotte, & Lefebvre, 2008). Gephi software enables a modularity to detect communities within the network (see Khokhar, 2015, pp. 88–95). The modularity value describes whether the studied network is arranged in communities of highly connected nodes (Fig. 2). *A resolution* in the modularity count is a value used for optimizing the number of communities, which means that a lower resolution is used to achieve more communities, whereas conversely, a higher resolution can be used to achieve fewer communities. The resolution can be understood as the sensitivity of the modularity, as in a multiplication with $\frac{1}{n}$, where $n \geq 1$ gives fewer communities and $n \leq 1$ gives more communities. For example, if communities are too large, the resolution can be used to optimize their size (Fortunato & Barthelemy, 2007). The present study used a standard resolution of 1.0, which means that communities were not manipulated (Lambiotte, Delvenne, & Barahona, 2009).

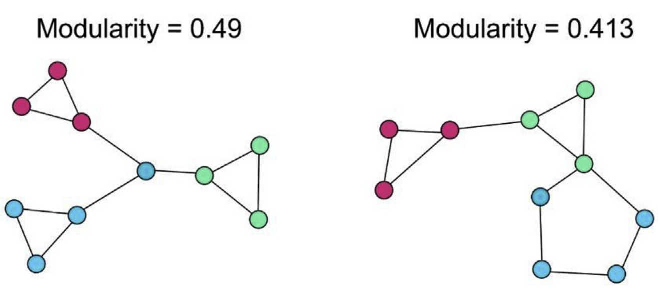
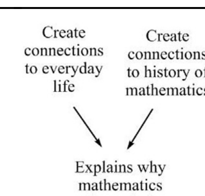
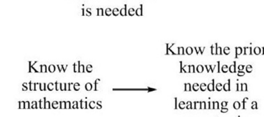
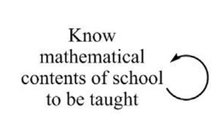


Fig. 2. The left network has three communities and high modularity, and the right network has the same number of nodes, edges and communities, but lower modularity (Griffiths, Pedersen, Fenton, & Petchey, 2014).

Table 2
The knowledge issues required for teaching mathematics were classified into the various domains of MKT and the contents of the essays were converted into networks to enable later network analysis.

Sample section of essay	Data-based classification (Conventional Content Analysis)	Theory-based classification (Direct Content analysis)	Network
A teacher should make mathematics meaningful. A teacher should connect the contents they are trying to teach to everyday life and provide more information on the history of mathematics.	Explains why mathematics is needed Create connections to everyday life Create connections to the history of mathematics	Specialized Content Knowledge Specialized Content Knowledge Specialized Content Knowledge	
Teachers need knowledge of the structure of mathematics, because they need to know what prior knowledge the new learning requires.	Know the structure of mathematics Know the prior knowledge in the learning of a new topic	Horizon Content Knowledge Horizon Content Knowledge	
Self-evidently, subject knowledge plays a major role. You cannot teach mathematics if you do not know mathematics. At school level, there are so many different topics, and a teacher needs to know them and use this knowledge.	Know mathematical contents to be taught in school	Common Content Knowledge	

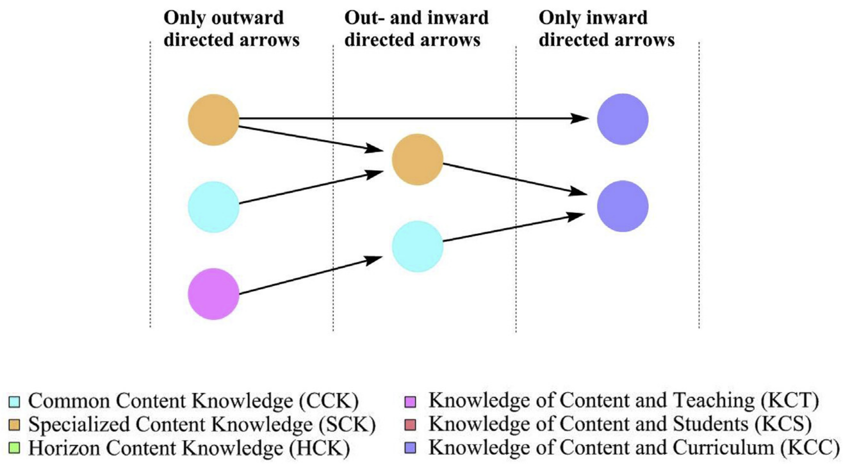


Fig. 3. The nodes in each section are organized into three regions and the colors show how the node is categorized according to MKT. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

In the present study, the term *domain* refers to theoretical knowledge, and the term *section* is used in relation to our empirical findings. We do not use the terms community or modularity because these usually refer to specific mathematical methods. However, as the large network represents teacher knowledge based on future teachers' ideas, and these network communities represent the various sections of teacher knowledge, for the sake of clarity, we will present the sections of teacher knowledge one by one in results section 4.2. For this purpose, we used the Gephi *Filter Tool* to filter out the other sections (see Khokhar, 2015, pp. 146–150). This filter tool removes all the nodes and arrows of the selected sections, leaving only the *internal* arrows of the relevant section. Furthermore, to represent each section in the same way, we arranged the nodes of each section in the order shown in Fig. 3. As the nodes were organized into three regions based on how the inward and outward directed arrows were attached to the nodes, these sections better describe *what kind* of knowledge is required for teaching mathematics (the left region), *why* this knowledge is needed (the middle region) and *in which context* this knowledge is needed (the right region). In each section, the color of the nodes refers to their theory-based classification: Thus the colors help describe the kind of knowledge involved in each section.

To find answers to the second research question concerning how the sections of teacher knowledge are connected to each other, we used the Gephi *Atlas2* algorithm to manipulate the network layout. The Atlas2 algorithm separates weakly connected parts, but also keeps strongly connected parts together (Jacomy, Venturini, Heymann, & Bastian, 2014). Because the modularity count identifies communities and Atlas2 manipulates the layout to show how the various communities are related, it is reasonable to use both to explore the ways in which the communities are settled in the network (see Khokhar, 2015, pp. 235–238). We will present the network representing all the teacher knowledge sections in results section 4.1.

In order to obtain more detailed answers to the second research question, we also examined the hierarchy of nodes in the network. A *partition parameter* can identify the properties of nodes or arrows in Gephi that are similar. In our case, all the nodes were classified into the six domains of MKT, hence this information could be used in Gephi as a partition parameter. First, the partition parameter marks nodes that were categorized as belonging to the same MKT domain in the same color (Khokhar, 2015, pp. 87–96). Next, we used a *grouping tool*[2] to examine the ways in which the six colors (domains of MKT) were connected to each other (Khokhar, 2015, pp. 209–211). They way in which the six MKT domains were interconnected will be shown in results section 4.3. As an aid to reading the figures in the results section, we provide an explanation of the sizes and colors of the nodes, arrows, and loops in Table 3.

4. Results

The findings will be presented in the following way. First, we will discuss how the empirical sections, identified using Gephi software, are interconnected and form the larger network. Then we will discuss each empirical section of the network separately and compare it with the six MKT domains. Finally, we will discuss the structure and hierarchy of the network.

4.1. Teacher knowledge sections within the main network

The future teachers' essays on *The kind of knowledge needed for teaching mathematics* were converted into a single large network. The Gephi software detected eighteen different teacher knowledge sections (modularity 0.601). Because the Gephi Atlas2 algorithm software separates weakly connected parts and keeps strongly connected parts together, sections that have more likely connections are considered "neighbors". Fig. 4 represents how the identified sections of teacher knowledge are interconnected and form the large network.

Fig. 4 shows, for example, that the *Knowing mathematics* section is related to the *Linking mathematics* section. This makes sense, as teachers first need to know mathematics before they can link mathematics to other areas. *Knowing the common challenges in mathematics* also bears a relationship to, for example, *Separating teaching* and *Choosing appropriate teaching methods*.

Interestingly, almost all the sections were linked to *Choosing appropriate teaching methods* and *Promoting students' different ways of learning* (see Fig. 4). These two sections, which were strongly related to each other, formed the center of the network. From this perspective, the knowledge required for teaching mathematics appeared to find its focus in two questions in the future teachers' minds: "How can we select appropriate teacher methods?" and "How can we promote different ways of learning?". Interestingly, *Following the national curriculum guidelines* had no links to the other sections. This suggests that the future teachers expressed

[2] This option exists in the Gephi 0.8.2. version, which we used.

Table 3
Explanation of sizes and colors of nodes, arrows, and loops in the Figures.

In network	Represents
Node	The kind of knowledge issue the future teachers discussed. Data-based classification.
Node size	How many arrows penetrated the node. Based on the in-degree* value of each node according to Gephi.
Node color	To which MKT domain the knowledge issue under discussion was related. Based on theory-based classification. This concerns all the Figures except Fig. 4, in which the node colors indicate teacher knowledge sections.
Arrow	How the knowledge issues under discussion were related to each other. Data-based classification.
Loop	Future teachers mentioned this knowledge issue, but omitted any understanding of its specific purpose. Data-based classification.
Arrow size	How many future teachers mentioned this relation. Based on edge weight value according to Gephi.
Arrow color	Which MKT domain the current arrow indicated. Based on the color of the Gephi target node.

* *In-degree* refers to the number of inward-directed arrows attached to each node whereas *out-degree* refers to the number of outward-directed arrows from each node.

issues related to the contents of the national curriculum and other issues in this section as separate entities. Fig. 4 provides an overview of the sections and their internecine arrangement, but the most strongly connected individual knowledge topics were located inside of each section. Therefore, we will next discuss each section of teacher knowledge separately.

4.2. Teacher knowledge sections

Fig. 4 shows that ten of the sections were interconnected, while eight sections were smaller and separate from the others. The eight separate sections only had one to four nodes, and were thus combined and presented as a single section (Section 11). In this section, the colors in Figs. 5–15 show how the node is categorized according to MKT.

4.2.1. Knowing mathematics

The future teachers considered that teachers required knowledge of mathematical theories, terms, rules, concepts and symbols (Fig. 5). Fig. 5 shows that these are useful knowledge for discussing mathematics, advising students during lessons, justifying and proving mathematical theorems, applying mathematics, evaluating how exercises work in practice and inspiring students. In this section, the issues on the left are background knowledge and are related to *Knowing Mathematics* in general, but they are also connected to issues related to the domains of KCT, CCK, SCK and KCS.

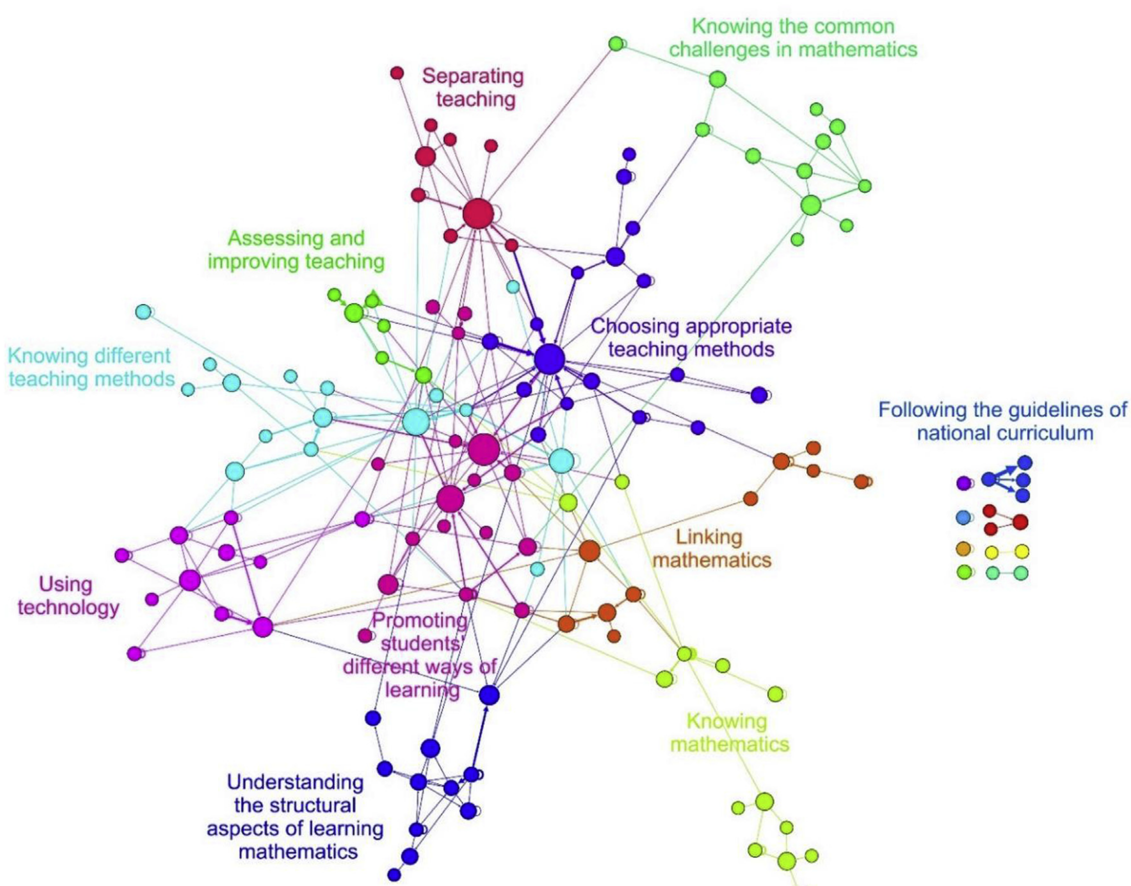


Fig. 4. How the various teacher knowledge sections (=colors) are related to each other. Layout produced using Atlas2 algorithm. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

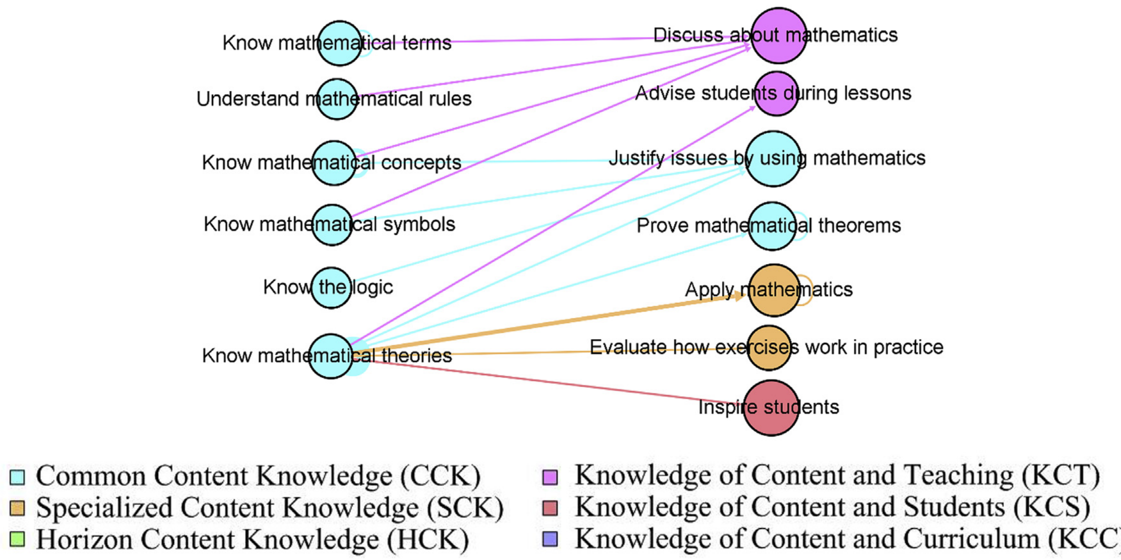


Fig. 5. Mathematical theories are needed for proving, discussing and applying mathematics.

4.2.2. Linking mathematics

According to the future teachers' views, teachers connect mathematics to other school subjects, everyday life and the history of mathematics when explaining why mathematics is needed, when answering students' questions, and when promoting students' understanding (Fig. 6). However, teachers should know mathematics and be able to explain it in many different ways in order to promote students' understanding. In this section, many issues are related to creating connections or producing a storyline around mathematics, hence we have called it *Linking mathematics*.

4.2.3. Understanding the structural aspects of learning mathematics

The future teachers pointed out that teachers need to understand the hierarchy of mathematics and to know the structure of mathematics because they need to possess the prior knowledge needed for learning a new topic and for examining what students know at that point (Fig. 7). Furthermore, they considered that understanding the structure of mathematics and possessing prior knowledge was important, because teachers need to construct new mathematical knowledge on the foundations of their prior knowledge and need to teach how mathematics is constructed. In addition, filling in the gaps in students' knowledge requires knowledge of what the students have previously learned and how mathematics is constructed. All these issues are related to understanding the *structural aspects of learning mathematics*, which connects almost all of them in this section to the definition of HCK.

4.2.4. Knowing the common challenges in mathematics

The future teachers felt that teachers need to help their students overcome difficulties in mathematics, present alternative solutions, and choose suitable exercises (Fig. 8). To achieve these aims, teachers require background knowledge that will enable them to recognize students' mathematical ideas or inaccuracies in their reasoning. In this context, the future teachers mentioned the pitfalls of mathematics that hinder learning, rather than learning difficulties in general. In this particular section, the future teachers most frequently discussed *knowing the general challenges in mathematics*, which in this case were related to SCK.

4.2.5. Knowing different teaching methods

The future teachers claimed that in order to be able to control the interaction between students as well as that between students and teachers, teachers should present suitable questions and

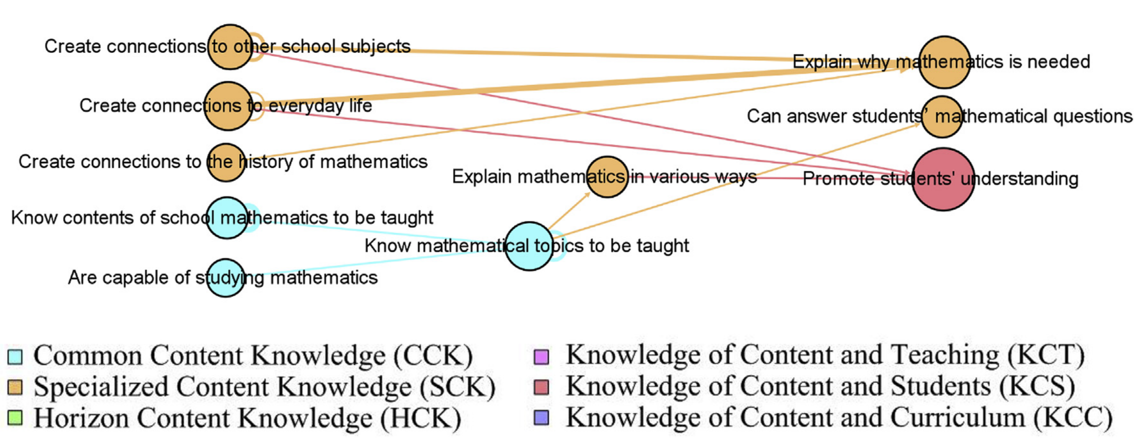


Fig. 6. Connections should be created between mathematics and everyday life that explain why mathematics is needed.

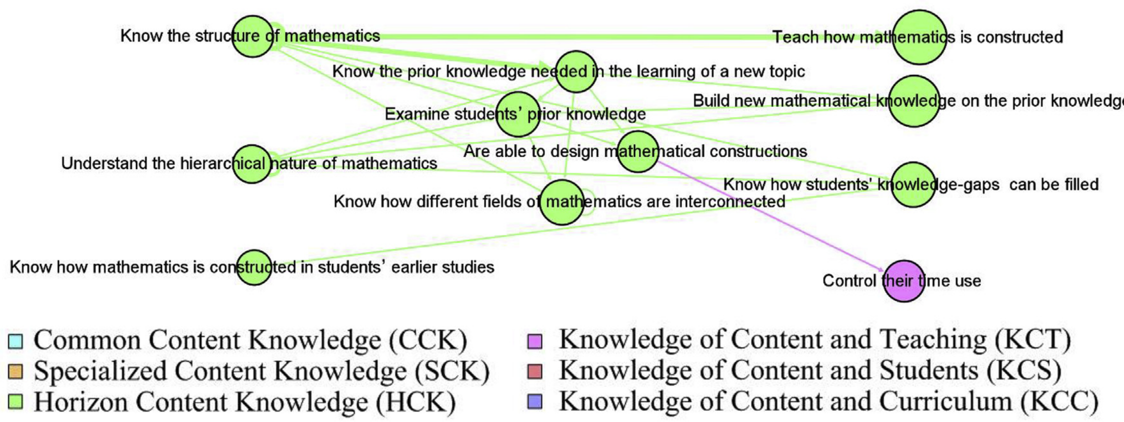


Fig. 7. Structural understanding of mathematics is necessary for constructing new mathematics on the foundation of students' prior knowledge.

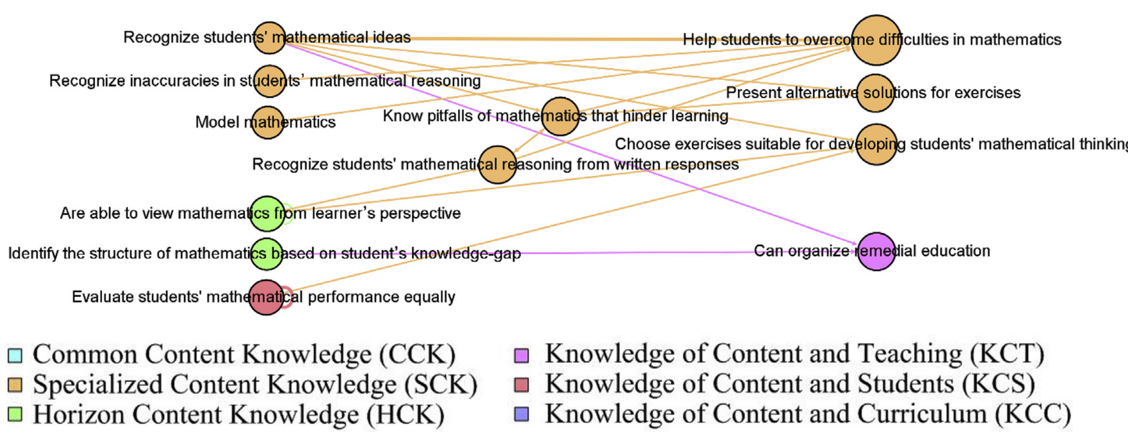


Fig. 8. Awareness of the general challenges posed by mathematics is necessary for teachers to help students learn mathematics.

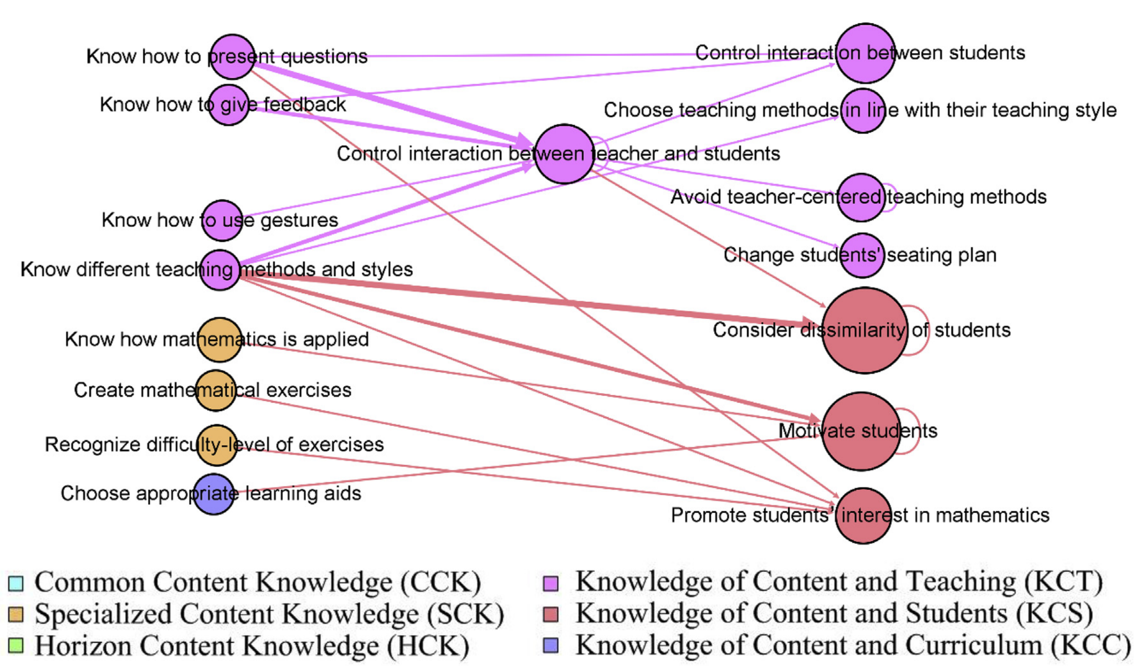


Fig. 9. Different teaching methods are needed in order to motivate students and to control interaction.

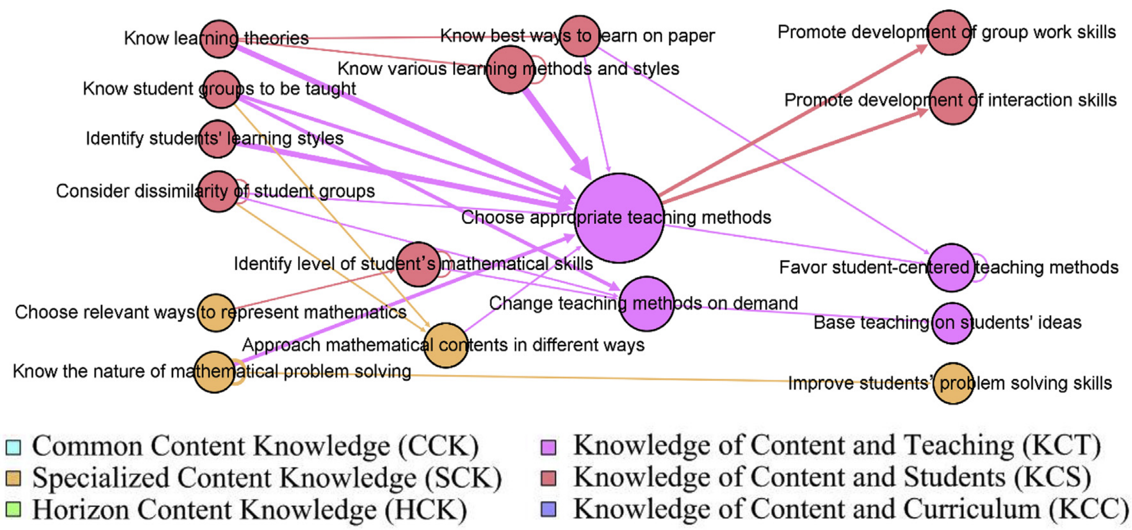


Fig. 10. Theoretical and practical knowledge related to learning are prerequisites for choosing an appropriate teaching method for each teaching situation.

provide feedback (Fig. 9). Avoidance of teacher-centered teaching methods or changing a seating plan can help control the interaction between students and the teacher. In addition, knowing different teaching styles is related to motivating students, considering the dissimilarities between them, and promoting their interest in mathematics. Furthermore, knowing how mathematics is applied, creating mathematical exercises, and recognizing the difficulty level of exercises are all related to motivating students and promoting their interest in mathematics. *Knowing different teaching methods* seems to be "an intersection node" in this section, which is linked to the domains of KCT and KCS.

4.2.6. Choosing appropriate teaching methods

According to the future teachers, teachers require theoretical knowledge about learning that will enable them to recognize the best ways in which to learn as well as the range of available learning methods before they can choose the most appropriate teaching methods for a particular teaching situation (Fig. 10). In addition, teachers need to know their student groups, to identify their students' learning styles and to consider the dissimilarities between student groups before they can choose appropriate teaching methods. Conversely, the future teachers felt that, before an appropriate teaching method can be selected, teachers need knowledge in order to be able to choose the most relevant ways of presenting mathematics and approaching mathematical content. This section shows how various issues are closely connected with the choice of appropriate teaching methods (the in-degree of the node, see Table 3). *Choosing appropriate teaching methods* seems to require background knowledge related to the domains of KCS and SCK.

4.2.7. Promoting students' different ways of learning

According to the future teachers, in order to support the dissimilarities between students' ways of learning in their own

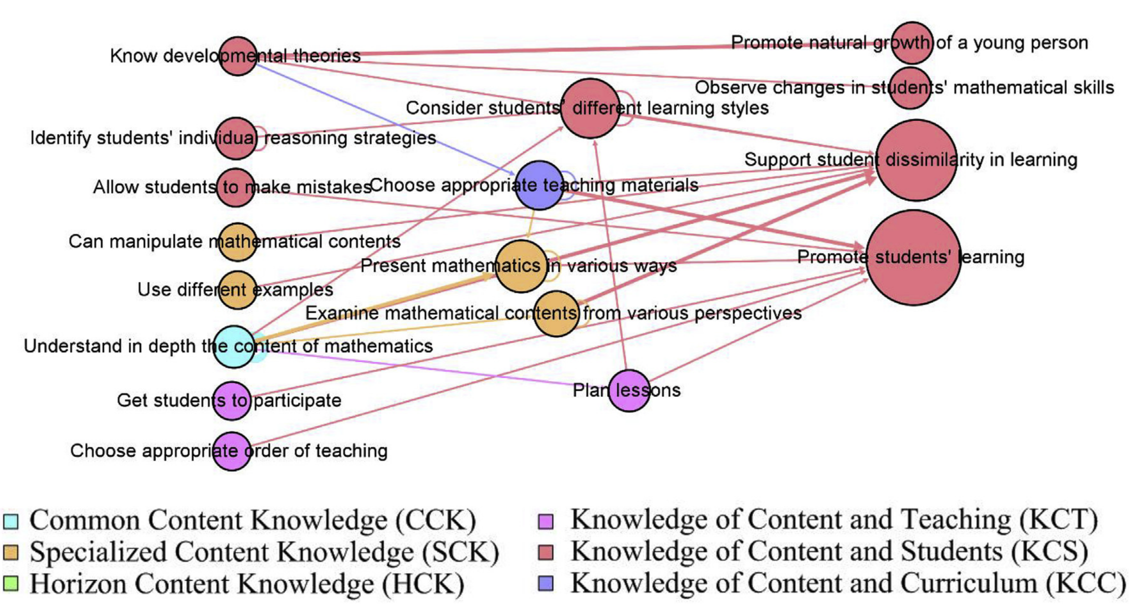


Fig. 11. Developmental and mathematical theories are needed to support students' dissimilarities and promote their learning.

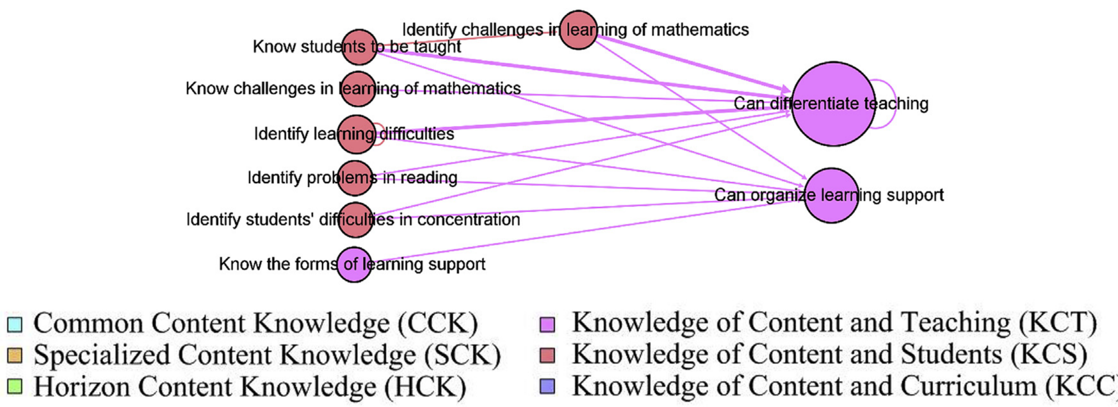


Fig. 12. Identification of learning challenges is necessary for differentiating modes of teaching.

teaching, they must take students' learning styles into account, and choose appropriate teaching materials, present mathematics in various ways, examine mathematics from various perspectives, use different examples and manipulate the mathematical contents (Fig. 11). Generally speaking, the same issues are connected to promoting students' learning. Choosing an effective teaching sequence, getting the students to participate, and designing a good lesson plan are also linked to promoting students' learning. Since most of these issues are connected to promoting students' learning and taking students' dissimilarities into account, we named this section *promoting students' different ways of learning*. These issues were classified under KCS, because in this particular case, the future teachers discussed their evaluation and understanding of students' behavior.

4.2.8. Separating teaching

The future teachers considered that, in addition to teachers needing to know their students, they also need to recognize the general challenges that students may face in learning mathematics, as well as their learning difficulties and potential problems in terms of reading skills and concentration (see Fig. 12). In this case, the future teachers devoted some attention to the general difficulties faced in learning mathematics. This kind of knowledge is yet another form of background knowledge for differentiating teaching and organizing learning support for students. In this section, the observations related to students can be identified as KCS, although actions taken with regard to *Separating teaching* and organizing learning support are related to KCT.

4.2.9. Assessing and improving teaching

According to the future teachers' views, mathematics teachers should be able to evaluate their own teaching, also that based on student feedback (Fig. 13). Furthermore, teachers need to be aware of educational trends and possess research-based knowledge of learning mathematics. Since all these issues are directed toward improving teachers' own teaching, we named this section *Assessing and improving teaching*.

4.2.10. Using technology

The future teachers were of the opinion that a teacher needs to master the use of technological equipment such as calculators, software, applications, and other concrete learning aids (Fig. 14). This knowledge is required for choosing appropriate technological aids, software, and other learning aids to make teaching interesting and diverse. In addition, the future teachers felt that technology could be used to illustrate the special characteristics of mathematics. This section relates to *Using technology,* although some of its issues connect the KCC domain to those of KCT and SCK.

4.2.11. Following the guidelines of the national curriculum

The future teachers believed that a teacher needs to know the contents of their national curriculum in order to be able to follow its guidelines, to know what they should teach and to enable their students to study according to the stipulations of the curriculum (Fig. 15). In addition, teachers must search for new exercises, for example on the internet, and collect their own bank of teaching-related materials, which in turn will help them select appropriate exercises for their teaching. Knowing pedagogical concepts helps in discussions on teaching with colleagues and displaying an interest in mathematics can make a significant difference in promoting the individual teacher's own mathematical proficiency. Based on the thickness of the arrows (i.e., the number of future teachers, see

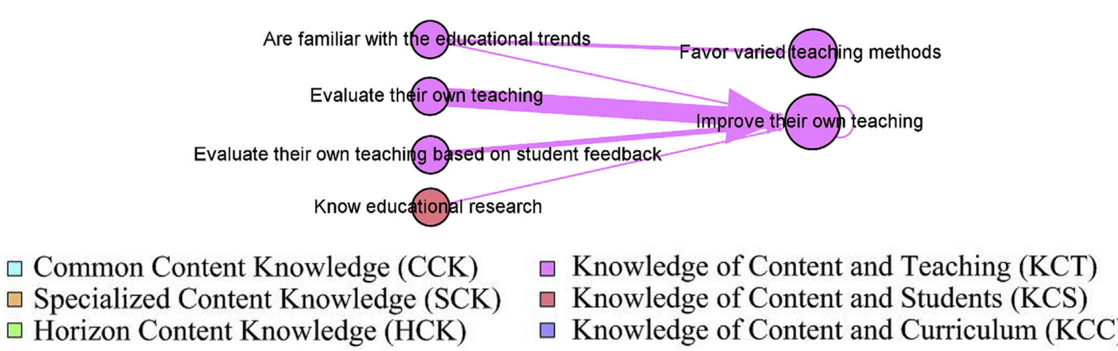


Fig. 13. Self-evaluation skills are necessary for improving teaching.

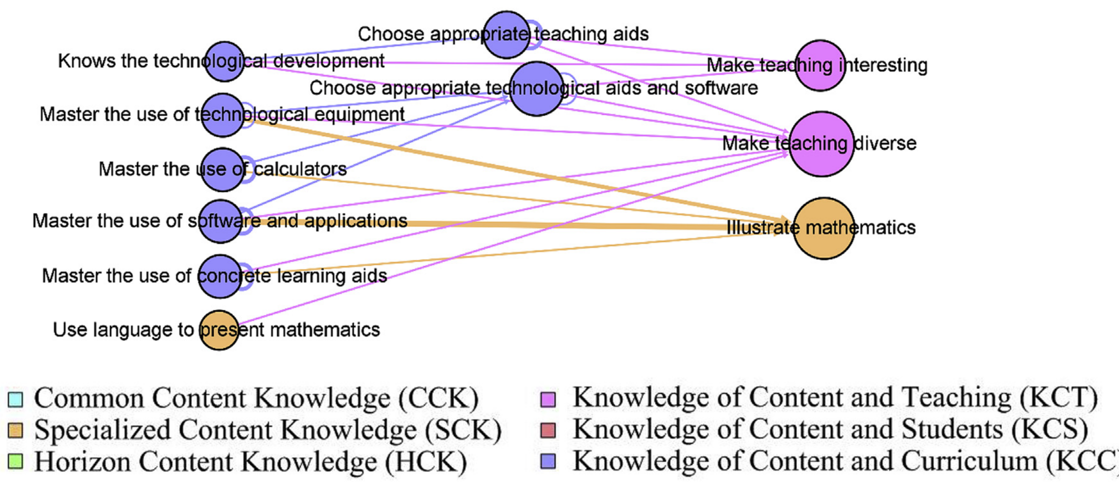


Fig. 14. Technological aids are necessary for illustrating mathematics.

Table 3), this section relates most closely to *Following the guidelines of the national curriculum.*

4.3. Structure of teacher knowledge

Since all of the 136 nodes were classified into the six domains of MKT, we used the Gephi Grouping tool to investigate how these domains were interconnected. The result is presented in Fig. 16.

Fig. 16 reveals that CCK has an out-degree total of 30 and an in-degree total of zero. This suggests that all the issues related to CCK in the minds of future teachers are background knowledge related to doing something. Similarly, KCC has an out-degree total of 37 and an in-degree total of only three. According to the results, both CCK and KCC represent background knowledge for the other knowledge domains, thus we identified them as *Foundation knowledge* for teachers (connections mostly outside).

The in-degree and out-degree totals are in balance for the domains of HCK and SCK because in the case of these two domains, the same numbers of arrows are directed inside and outside. Since all of the knowledge topics (nodes) are classified into the MKT domains, Figs. 5–15 are synchronized with Fig. 16. Therefore, the results presented in Sections 4.2 and 4.3 are also in harmony. The result presented in Section 4.2 shows that the future teachers believe that teachers need background knowledge of mathematical theories (CCK) in order to be able to draw up connections between mathematics and everyday life, other school subjects and the history of mathematics (SCK). In general, the future teachers seemed to think that teachers convert the content of their subject into a form that will help students understand why and where mathematics is needed. In this phase, teachers transform subject knowledge from its form as it exists in CCK into a form of SCK. Similarly, the future teachers thought that a teacher needs to master the various uses of technology and other teaching aids, which we can classify as familiarity with KCC. They also thought that teaching aids, especially technology, could be used to, for example, illustrate mathematics. The same transformation happens in this phase. The process begins with mastering teaching aids and continues to the selection of suitable ways to use the aids. In order to illustrate mathematics knowledge itself, teachers must have knowledge of the particular characteristics of mathematics.

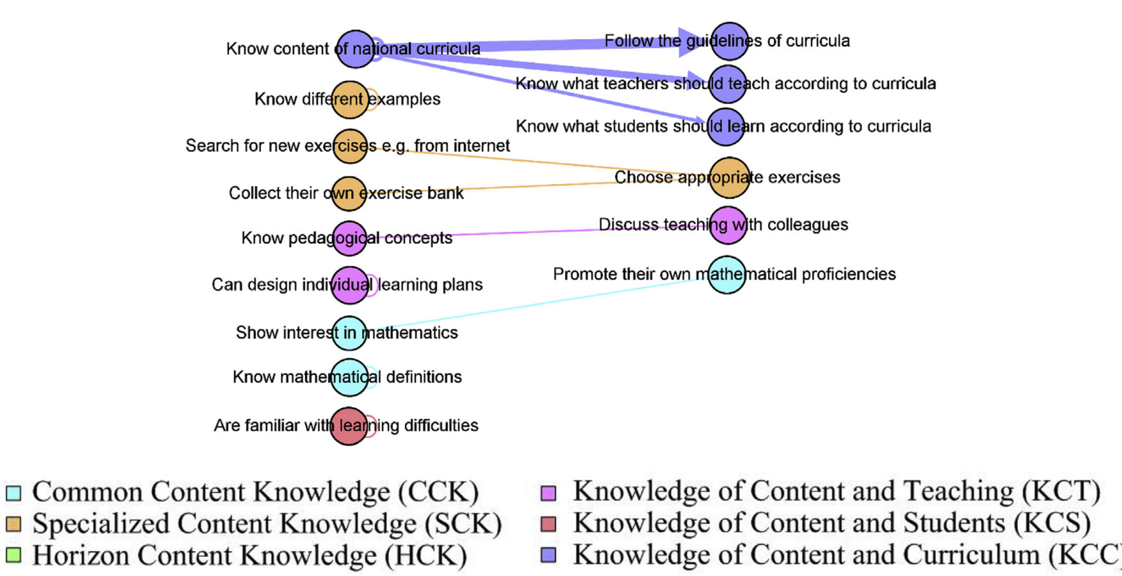


Fig. 15. Knowing the contents of the national curriculum is a prerequisite for following its guidelines.

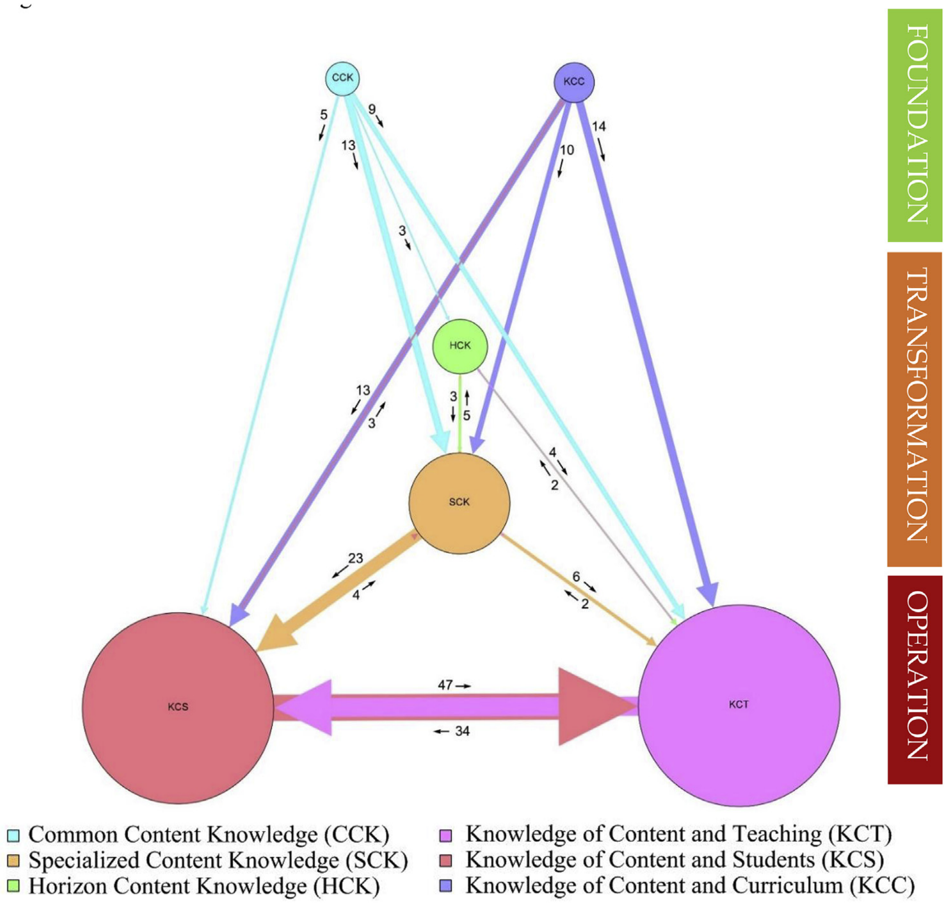


Fig. 16. The result demonstrates the hierarchical order of the six domains of MKT. The node sizes are based on their in-degree value.

According to the results, the domains of HCK and SCK are related to changing the form of the knowledge, and hence we named these domains *Transformation knowledge* for teachers (in- and out-connections in balance).

According to the results represented in Fig. 16, the domains of KCS and KCT are the final phase in the totality of teacher knowledge. Both types of Foundation and Transformation knowledge have been consigned to these two domains. The domains of KCS and KCT are also strongly connected to each other. This makes sense, as the results presented in Section 4.2 show that teachers need to know learning theories and make empirical observations about their students and that both of these are effective starting points for the selection of appropriate teaching methods. Because teachers use their knowledge of how students learn (in theory and in practice) to make decisions about how to teach, the domains of KCS and KCT are linked together. Teachers may recognize the challenges inherent in student learning or they may note that students struggle in learning mathematics (in this case, SCK would also be involved), which has an effect on the kind of decisions that the teacher will make in these situations. According to the results, decisions that include the selection of appropriate teacher methods and of promoting students' different ways of learning are related to almost all the sections, and hence the two questions, "How can we select appropriate teacher methods?" and "How can we promote different ways of learning ?" also activate all the domains of MKT. According to the results, the KCS and KCT domains are best related to actual teaching events, and hence we named this domain *Operation knowledge* for teachers (connections mostly inside).

5. Discussion

According to the MKT framework, recognizing students' incorrect answers falls under CCK and identifying the nature of an error comes under SCK, whereas knowledge of typical errors is an aspect of SCK (Ball et al., 2008). In classroom situations, the types of knowledge related to the six MKT domains are interconnected. However, at present, the MKT "egg model" is unable to describe the domain relations or the structure of teacher knowledge (see Fig. 1). Since the research understanding of how domains influence each other is limited, we cannot fully understand the role played by a single MKT domain in relation to the totality of teacher knowledge.

In this article, we have demonstrated that network analysis may offer new tools for investigating the domain interactions and structure of teacher knowledge. Because our sample of eighteen future teachers is limited, our results cannot be generalized. However, the network analysis methods may offer new answers within this research field, and a network may itself be an instructive way in which to describe teacher knowledge (see Figs. 4 and 16). Hence, our findings have encouraged us to argue that in studying future teachers' perceptions of *what kind* of knowledge is needed for teaching (nodes), it is equally important to study *how* knowledge topics are interconnected in the minds of teachers (arrows). As the sections of teacher knowledge are based on how strongly knowledge components are interconnected, each section describes not only *what kind* of knowledge is needed but also *why* and *in which context* the knowledge is needed. The fact that the nodes and arrows can both be included in the same data and simultaneously analyzed also enables us to study the structure and hierarchy of teacher knowledge.

5.1. Domain interactions and structure of teacher knowledge

Our most interesting finding was that the six MKT domains exist in a hierarchical sequence in the minds of future teachers. The future teachers were of the opinion that CCK and KCC are more like background knowledge, and are needed for doing something. Because of the hierarchy, the result indicates that to understand SCK and HCK, a teacher should first acquire CCK and KCC. This makes sense, because to understand the structure of mathematics (HCK), teachers should have broad knowledge of several mathematical contents (CCK). Teachers also need to know how the contents are organized in the national curriculum (KCC) in order to understand the most effective sequence in which mathematics can be presented to students (HCK). In addition, teachers require a deep understanding of different mathematical theories (CCK) in order to be able to present mathematics, to demonstrate relevant examples, to create connections, and to apply the mathematics (SCK). According to the results, all four domains (CCK, KCC, SCK, and HCK) are related in different ways to the last two domains. Thus, these last two domains (KCS and KCT) require the most background knowledge. These results show that, at least in the minds of the future teachers investigated here, an MKT hierarchy exists (see Table 4).

The hierarchy highlights that pure mathematical knowledge is a base for teacher knowledge. In addition, in the Knowledge Quartet, mathematical knowledge and beliefs refer to Foundation Knowledge, whereas each of the other three domains is based on Foundation Knowledge (Rowland et al., 2009). Likewise, in the Ladder of Knowledge, subject matter knowledge comes first, followed by pedagogical knowledge. Knowledge of effective teaching comes last (O'Meara, 2011). O'Meara (2011, p. 31) suggested that "*Teachers must combine their knowledge of general mathematical concepts and applications as well as their pedagogical knowledge and convert it into representations, explanations and analogies that students will understand and appreciate*". Kleickmann et al. (2015, p. 123) found evidence that teachers need Content Knowledge (CK) to understand Pedagogical Content Knowledge (PCK), but propose that "*[...] beyond a certain threshold level, additional CK may no longer be conducive to the development of PCK. However, this suggestion is speculative and requires further investigation*".

Many teacher knowledge frameworks respond to the question of *what kind* of knowledge is needed for teaching mathematics, but they rarely describe *how* domains are interconnected or how knowledge is structured (e.g. Baumert & Kunter, 2013; Shulman, 1986). It is logical that some knowledge is needed before new topics can be understood. However, the question of how teacher knowledge is structured also relates to the question of how the contents of mathematics teacher education should be scheduled. Should teachers first acquire a knowledge of mathematics and then a knowledge of pedagogy to understand what effective teaching involves? Some studies support this approach of scheduling the contents of mathematics teacher education (e.g. O'Meara, 2011). Wu (2005, p. 6) claimed that learning pedagogical content knowledge without first learning mathematical knowledge is the same as asking teachers to run before they can walk. However, if teacher knowledge contains hierarchical sequences and some knowledge is needed before new topics can be understood, learning the latter parts of teacher knowledge would require more background knowledge, which means that understanding these parts may be more challenging. If we assume that this is true, our results indicate that future teachers may consider understanding KCT and KCS the most challenging. These two MKT domains are connected in many different ways to the other four MKT domains, and therefore, understanding KCT and KCS requires also knowing the four other MKT domains.

There seem to be less consensus about the hierarchy of teacher knowledge, how domains influence each other, or how teacher knowledge is structured (e.g. Hashweh, 2005; Shulman, 1986). Baumert and Kunter (2013, p. 28) also argued that "*there is far less agreement about the structure of this [teacher] knowledge, the different types of knowledge and their epistemological status, or the development and mental representation of professional knowledge and skills*". If the research understanding of how domains influence each other is limited, we cannot fully understand the role played by a single domain in relation to the totality of teacher knowledge. If the nature of teacher knowledge is hierarchical, understanding the middle parts of teacher knowledge might be critical for understanding the totality of teacher knowledge.

Our findings indicate that SCK and HCK may be good candidates for the role of "middle part" or "critical" teacher knowledge. If we omit these two domains in teacher education, the necessary totality of teacher knowledge may very well collapse (see Fig. 16). If we remove Transformation knowledge (HCK and SCK), then most of the links between Foundation knowledge (CCK and KCC) and Operation knowledge (KCS and KCT) disappear. This could lead to teachers finding that although they have learned mathematics and pedagogy, they cannot connect these two knowledge types properly in their minds. This problem is related to the challenge of transforming theoretical knowledge into the practice of teaching (e.g. Korthagen, 2010; Korthagen & Kessels, 1999). When teachers feel that they cannot connect mathematics and pedagogy, some of the benefits of their teacher knowledge will undoubtedly disappear. Therefore, in broad terms, we can say that our finding means that knowing CCK is insufficient for teaching, as this knowledge must first be transformed into SCK until mathematical knowledge becomes activated in different teaching situations. Copur-Gencturk and Lubienski (2013, p. 219) also claimed that "*perhaps specialized content knowledge should actually be considered a subset of common knowledge if it simply adds a teaching-specific story line around the everyday mathematics content*".

If teacher knowledge is interpreted as a network, we can analyze in several ways how different components of teacher knowledge influence each other, using network analysis methods. Since the *sections* of teacher knowledge itself are identified on the basis of how strongly they are connected, each section describes *what kind* of knowledge is needed for teaching, *why* knowledge is needed and *in which context* this knowledge is needed. Furthermore, sections

Table 4
The hierarchy of the network suggests that the MKT domains can be categorized into three different types of knowledge.

Type of knowledge	Domains of MKT
Foundation knowledge (*Connections mostly outside*)	Common Content Knowledge (CCK) Knowledge of Content and Curriculum (KCC)
Transformation knowledge *(Inside and outside connections in balance)*	Horizon Content Knowledge (HCK) Specialized Content Knowledge (SCK)
Operation knowledge *(Connections mostly inside)*	Knowledge of Content and Students (KCS) Knowledge of Content and Teaching (KCT)

that are more likely to be connected are considered "neighbors" in the large network (see Fig. 4). In our case, two sections in the middle of the large network are strongly connected to other sections. These two domains are simultaneously related to several sections of teachers' knowledge, and therefore questions such as "How can we select appropriate teacher methods?" and "How can we promote different ways of learning?" are indirectly related to all sections of teacher knowledge.

5.2. Contribution, limitations and further ideas of this study

The principal contribution of the present study is the proposal of a new network approach to investigating the kind of knowledge required for teaching. Further, it looks at the ways in which knowledge topics are interconnected, and how teacher knowledge is structured in the minds of future teachers. In this respect, our findings fill a gap in this area of research on teacher knowledge. Two recently published review studies suggest that the investigation of the nature, impact and development of teacher knowledge constitutes the three mainstreams of research on teacher knowledge (see Depaepe, Verschaffel, & Kelchtermans, 2013; Hoover, Mosvold, Ball, & Lai, 2016). Investigating the relationship between subject matter knowledge and pedagogical content knowledge is one of the lines of research in the first mainstream (Depaepe et al., 2013). It is noteworthy that all but one of the studies reviewed were ascertaining studies (Depaepe et al., 2013). Generally speaking, small-scale studies aim at unraveling the relationship between teachers' subject matter knowledge and pedagogical content knowledge as enacted in the classroom, while large-scale studies more frequently use distinct test items to measure the relationship (Depaepe et al., 2013). Our own study fills a gap in this research field by presenting a new, explorative approach to investigating the interconnections in teacher knowledge.

We discovered some interesting aspects of the structure of future teachers' views regarding the knowledge required for teaching, and of the ways in which the knowledge components are interconnected and how teacher knowledge is structured. At this stage, however, our results cannot be generalized without further research. The data were collected at only one Finnish university, and consisted of future teachers' perceptions of the knowledge required for teaching mathematics. Given their relatively limited experience of school teaching, future teachers' understanding of the work of a teacher is limited. In consequence, generalizing our findings to other samples, time periods, or settings on the basis of our present results poses a challenge. But it is precisely for this reason that the approach should be applied again. It could be used, for example, to investigate the ways in which qualified teachers or teacher educators view teacher knowledge. The amount of data could be increased to saturation point, and the national or cultural similarities and differences of teacher knowledge could also be studied if data were collected internationally.

A large database encompassing teacher knowledge drawn from various subjects and school-levels would facilitate the use of network analysis methods to examine specific subject-related and school-level differences in teacher knowledge. Identifying the common and specific aspects of teacher knowledge may provide a key to understanding the similarities and differences inherent in teaching and learning different subjects.

The network approach presented here might be useful for also investigating other educational questions. However, this would only work when school subjects are explicitly structured, as is the case for mathematics. If we assume that data can be expressed as nodes and links between them (networks,) then network analysis algorithms can be used, and the findings can be set against theoretical understanding. Network analysis methods have already provided a new understanding of the internet, social networks, organizational networks, neural networks, metabolic networks, blood vessels, the structure of brains, food webs, and ecosystems (e.g. Bullmore & Sporns, 2009; Newman, 2006; Newman & Girvan, 2004; Supekar, Menon, Rubin, Musen, & Greicius, 2008). Thus, it is possible that network analysis methods may also provide us with a new understanding of teacher knowledge.

Acknowledgement

We would like to thank The Finnish Cultural Foundation, North Karelia Regional Fund for funding our research project, Evaluating and Improving Mathematics Teacher Education, with several grants in recent years (grant numbers 55131472, 55141384 and 55151942).

References

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*. Unpublished manuscript. Retrieved from https://eldorado.tu-dortmund.de/bitstream/2003/31305/1/003.pdf.

Ball, D. L., Thames, M. A., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Bastian, M., Heymann, S., & Jacomy, M. (2009). Gephi: An open source software for exploring and manipulating networks. In *Proceedings of the third international conference on weblogs and social media* (Vol. 8, pp. 361–362). Retrieved from http://aaai.org/ocs/index.php/ICWSM/09/paper/view/154/1009.

Baumert, J., & Kunter, M. (2013). The COACTIV model of teachers' professional competence. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 25–48). New York, the United States of America: Springer US.

Blondel, V. D., Guillaume, J. L., Lambiotte, R., & Lefebvre, E. (2008). Fast unfolding of communities in large networks. *Journal of Statistical Mechanics: Theory and Experiment, 2008*(10). https://doi.org/10.1088/1742-5468/2008/10/P10008. P10008.

Bullmore, E., & Sporns, O. (2009). Complex brain networks: Graph theoretical analysis of structural and functional systems. *Nature Reviews. Neuroscience, 10*(3), 186–198.

Cole, Y. (2012). Assessing elemental validity: The transfer and use of mathematical knowledge for teaching measures in Ghana. *ZDM, 44*(3), 415–426.

Copur-Gencturk, Y., & Lubienski, S. T. (2013). Measuring mathematical knowledge for teaching: A longitudinal study using two measures. *Journal of Mathematics Teacher Education, 16*(3), 211–236.

Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). "Mathematical knowledge for teaching": Adapting U.S. measures for use in Ireland. *Journal of Mathematics Teacher Education, 11*(3), 171–197.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12–25.

Dreher, A., Lindmeier, A., & Heinze, A. (2016). Conceptualizing professional content knowledge of secondary teachers taking into account the gap between academic and school mathematics. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 219–226). Szeged, Hungary: PME.

Dreher, A., Lindmeier, A., Heinze, A., & Niemand, C. (2018). What kind of content knowledge do secondary mathematics teachers need? *Journal für Mathematik-Didaktik, 39*(2), 319–341.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*(1), 13–33.

Fauskanger, J., Jakobsen, A., Mosvold, R., & Bjuland, R. (2012). Analysis of psychometric properties as part of an iterative adaptation process of MKT items for use in other countries. *ZDM, 44*(3), 387–399.

Fennema, E., & Franke, L. M. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York, NY: Macmillan.

Fortunato, S., & Barthelemy, M. (2007). Resolution limit in community detection. *Proceedings of the National Academy of Sciences, 104*(1), 36–41. https://doi.org/10.1073/pnas.0605965104.

Fung, D., Kutnick, P., Mok, I., Leung, F., Lee, B. P. Y., Mai, Y. Y., et al. (2017). Relationships between teachers' background, their subject knowledge and pedagogic efficacy, and pupil achievement in primary school mathematics in Hong Kong: An indicative study. *International Journal of Educational Research, 81*, 119–130.

Griffiths, E. C., Pedersen, A. B., Fenton, A., & Petchey, O. L. (2014). Analysis of a summary network of co-infection in humans reveals that parasites interact most via shared resources. *Proceedings of the Royal Society B: Biological Sciences, 281*, 20132286. https://doi.org/10.1098/rspb.2013.2286.

Hashweh, M. (2005). Teacher pedagogical constructions: A reconfiguration of pedagogical content knowledge. *Teachers and Teaching: Theory and Practice,*

11(3), 273–292.
Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.
Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406.
Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast, 13*(1–2), 3–34.
Hsieh, H. F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 1277–1288.
Jacomy, M., Venturini, T., Heymann, S., & Bastian, M. (2014). ForceAtlas2, a continuous graph layout algorithm for handy network visualization designed for the gephi software. *PLoS One, 9*(6), e98679. https://doi.org/10.1371/journal.pone.0098679.
Jankvist, U. T., Mosvold, R., Fauskanger, J., & Jakobsen, A. (2015). Analysing the use of history of mathematics through MKT†. *International Journal of Mathematical Education in Science & Technology, 46*(4), 495–507.
Jóhannsdóttir, B., & Gísladóttir, B. (2014). Exploring the mathematical knowledge of prospective elementary teachers in Iceland using the MKT measures. *Nordic Studies in Mathematics Education, 19*(3–4), 21–40.
Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of mathematical tasks of teaching and the corresponding LMT measures in the Malawi context. *The Mathematics Enthusiast, 13*(1), 171–186.
Khokhar, D. (2015). *Gephi Cookbook. Over 90 hands-on recipes to master the art of network analysis and visualization of Gephi*. Birmingham: Packt Publishing Ltd. Available from https://www.packtpub.com/big-data-and-business-intelligence/gephi-cookbook.
Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., et al. (2015). Content knowledge and pedagogical content knowledge in Taiwanese and German mathematics teachers. *Teaching and Teacher Education, 46*, 115–126.
Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2016). Teachers and their educators – views on contents and their development needs in mathematics teacher education. *The Mathematics Enthusiast, 13*(1–2), 149–170.
Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2017). How education affects mathematics teachers' knowledge: Unpacking selected aspects of teacher knowledge. *Eurasia Journal of Mathematics, Science and Technology Education, 13*(6), 1943–1980.
Korthagen, F. A. (2010). Situated learning theory and the pedagogy of teacher education: Towards an integrative view of teacher behavior and teacher learning. *Teaching and Teacher Education, 26*(1), 98–106.
Korthagen, F. A., & Kessels, J. P. (1999). Linking theory and practice: Changing the pedagogy of teacher education. *Educational Researcher, 28*(4), 4–17.
Kwon, M., Thames, M. H., & Pang, J. (2012). To change or not to change: Adapting mathematical knowledge for teaching (MKT) measures for use in Korea. *ZDM, 44*(3), 371–385.
Lambiotte, R., Delvenne, J. C., & Barahona, M. (2009). *Laplacian dynamics and multiscale modular structure in networks*. Retrieved from http://arxiv.org/pdf/0812.1770.pdf.
Landauer, T. K., Foltz, P. W., & Laham, D. (1998). An introduction to latent semantic analysis. *Discourse Processes, 25*(2–3), 259–284.
Markworth, K., Goodwin, T., & Glisson, K. (2009). The development of mathematical knowledge for teaching in the student teaching practicum. In D. S. Mewborn, & H. S. Lee (Eds.), *Scholarly practices and Inquiry in the preparation of mathematics teachers* (pp. 67–83). San Diego, CA: Association of Mathematics Teacher Educators.
Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017–1054.
Morris, A. K. (2006). Assessing pre-service teachers' skills for analyzing teaching. *Journal of Mathematics Teacher Education, 9*(5), 471–505.
Neubrand, M. (2018). Conceptualizations of professional knowledge for teachers of mathematics. *ZDM, 50*, 601–612.
Newman, M. E. (2006). Modularity and community structure in networks. *Proceedings of the National Academy of Sciences, 103*(23), 8577–8582. https://doi.org/10.1073/pnas.0601602103.
Newman, M. E., & Girvan, M. (2004). Finding and evaluating community structure in networks. *Physical review E, 69*(2), 026113, 1-15.
Ng, D. (2012). Using the MKT measures to reveal Indonesian teachers' mathematical knowledge: Challenges and potentials. *ZDM, 44*(3), 401–413.
Orrill, C. H., Kim, O. K., Peters, S. A., Lischka, A. E., Jong, C., Sanchez, W. B., et al. (2015). Challenges and strategies for assessing specialised knowledge for teaching. *Mathematics Teacher Education and Development, 17*, 12–29.
O'Meara, N. (2011). *Improving mathematics teaching at second level through the design of a model of teacher knowledge and an intervention aimed at developing teachers' knowledge*. Dissertation. Ireland: University of Limerick. Retrieved from https://ulir.ul.ie/bitstream/handle/10344/3254/OMeara_2011_teaching.pdf.
Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In T. Rowland, & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 9–25). Melbourne, Australia: Springer Science and Business Media.
Popping, R. (2003). Knowledge graphs and network text analysis. *Social Science Information, 42*(1), 91–106.
Popping, R., & Roberts, C. W. (1997). Network approaches in text analysis. In *Classification and knowledge organization* (pp. 381–389). Berlin, Heidelberg: Springer.
Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching: Reflecting on practice with the knowledge Quartet*. London, England: Sage publications Ltd.
Schmidt, W. H., Cogan, L., & Houang, R. (2011). The role of opportunity to learn in teacher preparation: An international context. *Journal of Teacher Education, 62*(2), 138–153.
Schmidt, W. H., Houang, R., & Cogan, L. S. (2011). Preparing future math teachers. *Science, 332*(6035), 1266–1267.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.
Speer, N. M., King, K. D., & Howell, H. (2015). Definitions of mathematical knowledge for teaching: Using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education, 18*(2), 105–122.
Supekar, K., Menon, V., Rubin, D., Musen, M., & Greicius, M. D. (2008). Network analysis of intrinsic functional brain connectivity in Alzheimer's disease. *PLoS Computational Biology, 4*(6). e1000100. Retrieved from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2435273/pdf/pcbi.1000100.pdf.
Tatto, M. T., Ingvarson, L., Schwille, J., Peck, R., Senk, S. L., & Rowley, G. (2008). Teacher education and development study in mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework. In *East lansing, MI: Teacher education and development international study center, college of education*. Michigan State University.
Tausczik, Y. R., & Pennebaker, J. W. (2010). The psychological meaning of words: LIWC and computerized text analysis methods. *Journal of Language and Social Psychology, 29*(1), 24–54.
Tchoshanov, M., Cruz, M. D., Huereca, K., Shakirova, K., Shakirova, L., & Ibragimova, E. N. (2017). Examination of lower secondary mathematics teachers' content knowledge and its connection to students' performance. *International Journal of Science and Mathematics Education, 15*(4), 683–702.
Tesch, R. (1990). *Qualitative research: Analysis types and software tools*. Bristol, PA: Falmer.
Wu, H. (2005). *Must content dictate pedagogy in mathematics education*. University of South Carolina. Retrieved from http://ed.sc.edu/ite/dickey/anaheim/Northridge2004c.pdf.