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Affect in Mathematics Education

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Definition

There are two different uses for the word “affect” in behavioral sciences. Often it is used as an overarching umbrella concept that covers attitudes, beliefs, motivation, emotions, and all other noncognitive aspects of human mind. In this article, however, the word affect is used in a more narrow sense, referring to emotional states and traits. A more technical definition of emotions, states, and traits will follow later.

From Anxiety and Problem Solving to Affective Systems

Mathematics is typically considered as the most objective and logical of academic disciplines. Yet, it has been widely acknowledged that mathematical thinking is not purely logical reasoning, but influenced much by affective features. The first systematic research agenda to study mathematics related affect was initiated within social psychology in the 1970s, focusing on mathematics anxiety as a specific branch of anxiety research. Anxiety is an unpleasant emotion of fear, which is directed towards an expected outcome in the future and it is often out of proportion to the actual threat. Surveys and meta-analyses (e.g. Dowker et al. 2016; Hembree 1990), show that mathematics anxiety is related to general anxiety, test anxiety, and low mathematics attainment. Also, female students have been found to be more prone to be anxious than male students, although they also seem to cope with their anxiety more efficiently than male students. Moreover, systematic desensitization has been confirmed to be effective in reducing mathematics anxiety.

An important distinction in anxiety research is that made between state and trait type of anxiety. A similar distinction is made also on a more general level between emotional (affective) state and trait. The emotional state refers to the emotion that arises in a certain situation, i.e., it is contextual and may change rapidly. On the other hand, a person’s rather stable tendency to experience certain emotional states across a variety of situations is called an emotional trait (Hannula 2012).

Within mathematics education, the research on problem solving notified the role of affect early on; already Pólya in his classical work (1957) mentioned hope, determination, and emotions. More explicitly the role of affect in mathematical problem solving was elaborated in several works published in the 1980s (e.g., Cobb et al. 1989; Schoenfeld 1985; for details, see McLeod 1992 or Hannula 2012). This literature on problem solving typically addressed the rapidly changing affective states in the dynamics of problem solving. Somewhat surprisingly, it was found out that in non-routine problem solving both experienced and novice problem solvers experience positive and negative emotions and that these emotions serve an important function in a successful solution process (e.g., Schoenfeld 1985; McLeod 1992).

Most of the research on mathematics-related affect by that time was summarized by McLeod (1992). He suggested a theoretical framework that has been influential in mathematics education research. He identified emotions, attitudes, and beliefs as the three major domains in the research of mathematics-related affect. Emotions were seen to be the least stable and most intense of the three, and to involve less cognition than attitude or beliefs. He also explicated the relationships between these categories in a theoretical framework: beliefs were seen as an element that influenced the initiation of emotions and repeated emotional reactions were seen as the origin of attitudes.

Also more recent research on affect in mathematics education emphasizes the relations between emotions and other affective variables (Hannula 2012). These include not only attitudes and beliefs but also values, motivations, social norms, and identity. The general trend is that a student who has a positive disposition towards mathematics tends to experience positive emotions more frequently and negative emotions less frequently than a student with a negative disposition. On the other hand, different theories (e.g., McLeod 1992) suggest that emotional experiences play a significant role in the formation of attitudes, beliefs, and motivation. Positive emotional experiences are seen as an important ingredient in the formation and development of a positive disposition. However, details are more complex than that. Some of the complexity is analyzed in a study (Goldin et al. 2011) that identified a number of behavioral patterns that integrate students’ affective and social interactions in mathematics classes.

Defining Emotions

In the literature, there are several definitions for emotions stemming from three distinct traditions: emotions as an outcome of evolution, psychoanalytic research, and cognitive tradition (cf. Hannula 2012). Yet, there is a general agreement that emotions consist of three processes: physiological processes that regulate the body, subjective experience that regulates behavior, and expressive processes that regulate social coordination. Moreover, most emotion theories agree that emotions have an important role in human coping and adaptation. Negative emotion (e.g., frustration) is experienced when progress towards a goal (e.g., solving a task) is prevented, and the emotion may suggest approaches (e.g., trying another tactic) to overcome the experienced conflict. Positive emotions, on the other hand, are experienced when progress is smooth. Emotions are an important part of memories and they will influence the choice of strategies in the future.

Emotion theories vary in the number of emotions they identify, the degree of consciousness they attribute to emotions, and the relation they perceive to exist between emotion and cognition (cf. Hannula 2011). Some theories identify a large number of emotions based on the diverse social scenarios and cognitive appraisals related to the emotion, while some other theories identify a small number of basic emotions (e.g., happiness, sadness, fear, anger, disgust, shame, and surprise) that differ in their physiology, and the different cognitive appraisals and social scenarios are seen as external (though closely related) to the emotion.

The Role of Emotional States in Self-Regulation

Emotions function on three different levels of self-regulation: physiological, psychological, and social (Power and Dalgleish 1997). The clearest example of the physiological adaptation is the “fight-or-flight” response to surprising threatening stimulus. Such physiological functions of emotions may have side effects that are relevant for learning. For example, the effects of high anxiety (fear) are detrimental for optimal cognitive functioning.

The psychological self-regulation of cognitive processing is an important function of emotions in any learning context, especially if we acknowledge the learner’s agency in the construction of knowledge. This function of emotions is deeply intertwined with metacognition. Empirical research has identified curiosity, puzzlement, bewilderment, frustration, pleasure, elation, satisfaction, anxiety, and despair to be significant in the self-regulation of mathematical problem solving (DeBellis and Goldin 2006). It is well established that emotions direct attention and bias cognitive processing. For example, fear (anxiety) directs attention towards threatening information and sadness (depression) biases memory towards a less optimistic view of the past (Power and Dalgleish 1997; Linnenbrink and Pintrich 2004). Although there is not yet sufficient evidence to conclude it for all emotions, it seems that positive emotions facilitate creative processes, while the negative emotions facilitate reliable memory retrieval and performance of routines (Pekrun and Stephens 2010).

As learning typically takes place in a social setting of the classroom, the function of emotions in the social coordination of a group is inevitably present. Most emotions have a characteristic facial expression, typically identifiable by movements in the brow region and lip corners. In addition, some emotions have specifically behavioral (e.g., slouching, clenched fists) or physiological (e.g., blushing, tears) expressions. Humans learn to interpret such expressions automatically and they form an important part of intrapersonal communication. Moreover, such visible expressions are more reliable than self-reported thoughts and feelings, which make emotions important observable indicators for related variables, such as goals, attitudes, or values. Perhaps the first to recognize the social emotions in mathematics education were Cobb et al. (1989) who identified students’ emotions to be related to two types of problems in collaborative problem solving: mathematical problems and cooperation problems.

The Spectrum of Emotional Traits

One of the recent developments is an increasing interest in the multidimensionality of mathematics-related emotions (Hannula et al. 2018, Liljedahl & Hannula 2016). Pekrun and his colleagues (2007) have developed a survey instrument to measure a number of achievement emotions, defined as emotions tied directly to achievement activities or achievement outcomes. Achievement-related activities are the origin of activity emotions (enjoyment, boredom, and anger). Outcome emotions include anticipatory emotions (hopelessness and anxiety) as well as emotions based on feedback (anger, pride, and shame). Another approach to multidimensionality of emotions has been Goldin et al. (2011) framework of archetypal affective structures.

Regulation of Emotions

Although emotions are functional for the human species, not all emotional reactions are functional in a learning context. For example, expert problem solvers seem to be controlling their emotions better than novices (e.g., Schoenfeld 1985). Emotion regulation refers to “the ways individuals influence which emotions they have, when they have them, and how they experience and express these emotions” (Gross 1998, p. 275). Few studies have addressed how students regulate their emotions in a mathematics class. A study by De Corte et al. (2011) suggests that active coping (i.e., effort), joking and acceptance, and social-emotional coping (i.e., seeking social support) as well as abandoning and negation are important strategies to reduce negative emotions or their effects.

The teacher can help students’ emotion regulation through modeling emotion regulation strategies or provide more direct support through controlling student emotions. Perhaps more effective than direct focusing on students’ emotion regulation is to develop the classroom climate. Feeling of community, an autonomy supportive teaching style, and an expressive environment have been found to support development of student emotion-regulation strategies (Fried 2012).

Creating an Emotionally Supportive Learning Environment

Although few studies have explored the individual strategies of emotion regulation, there is significant amount of general educational research on characteristics of a classroom that promote optimal emotional climate. Teacher enthusiasm, emphasis on mastery goals, positive feedback, optimal level of challenge, student autonomy and feeling of control, and meeting students’ needs can enhance positive student emotions (Pekrun and Stephens 2010).

Emerging empirical research indicates that classrooms are often emotionally flat, and boredom is one of the most frequently experienced emotions (Vogel-Walcutt et al. 2012). Schools have implemented programs to enhance students’ social and emotional learning in order to promote a healthy learning environment. Specific goals for these programs include competencies to recognize and manage emotions. According to a meta-analysis, such programs have beneficial effects on positive social behavior, problem behaviors, and academic performance (Durlak et al. 2011).

Cobb et al. (1989) emphasized the relationship between social norms and emotions. In their experimental classroom, the goal was engagement in mathematical activity, and therefore, even weaker students experienced and expressed positive emotions as they participated in group activities and whole class discussions. Mathematics educators have continued developing such learning environments that engage students emotionally, cognitively, and socially (Hannula 2018, Liljedahl & Hannula 2016). We know how to create such mathematics classrooms, but changing the reality of the classrooms on a large scale is a slow process.

Cross-references

Sociomathematical Norms in Mathematics Education

Mathematics Learner Identity

Teacher Beliefs, Attitudes, and Self-Efficacy in Mathematics Education

Gender in Mathematics Education

Values in Mathematics Education

Motivation in Mathematics Learning

Students’ Attitude in Mathematics Education

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