Affect and mathematical thinking: Exploring developments, trends, and future directions

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1. Affect and Mathematical Thinking Group - Looking back

The initial caution of the mathematics education community in considering affective issues is shown by the fact that CERME has hosted a thematic working group on affect only since its third meeting. Yet the first contribution on affect (Zan & Poli, 1999) had already appeared in CERME 1. Now the relevance of affect is recognised in the community of mathematics educators. In particular in ERME Conferences, the thematic working group (TWG) about affect and mathematical thinking has been consistently present, and increasingly well attended.

1.1 Development of theories in the field

The affect group in CERME has discussed extensively matters of conceptual framework and terminology, leading to more extensive theorisation of the area:

[The discussions] increased our awareness of being specific about the concepts that we use. We have realized that it is not sufficient to give definitions of the concepts that are being used in a particular study, but we have to explicate their relations to the other dimensions of affect research as well. (Hannula, 2011, p. 41)

There are three theoretical frameworks that have been influential in CERME for structuring the area of affect. The first is McLeod’s (1992) framework that identified three main topics of research in mathematics related affect: emotions, attitudes, and beliefs (Figure 10.1). Moreover, the framework suggested that emotions are the most intensive, the least stable, and the least cognitive of the three, while beliefs are at the other end of the continuum, and attitudes are in the middle.

[Insert Figure 10.1 here]

Figure 10.1: McLeod’s (1992) framework on Affect

A significant step forward was the graphic representation of the conceptual field that Peter Op ‘t Eynde composed during CERME 5 (Figure 10.2). This model captured new ideas discussed in the CERME affect group: it recognised motivation as an important concept and identified different levels of social context.

[Insert Figure 10.2 here]

Figure 10.2: The dimensions of mathematics-related affect and their relationships, presented at CERME 5 (Hannula, Op ‘t Eynde, Schlöglmann & Wedege, 2007, p. 204)

These ideas were further elaborated by Hannula in his CERME plenary (2011) and in an article for a CERME special issue of RME (2012). Hannula described three dimensions that can be used to identify and define affective theoretical concepts (Figure 10.3). The first dimension recognises that the concepts may be either cognitive (what one believes), affective (what one feels), or motivational (what one desires). The second, temporal dimension, separates state-type constructs that aim to describe dynamical processes from trait-type constructs that aim to describe rather stable dispositions. The third dimension recognised the social theories in mathematics education research, and also the embodied nature of affect, identifying three ontologically different traditions for affect-related research: psychological, social and embodied theories.

[Insert Figure 10.3 here]

Figure 10.3: Hannula’s (2011, p. 46; 2012) model of the dimensions for affective constructs

1.2 Shifting focus of attention

In order to empirically explore the relevance of different constructs and dimensions suggested above, we analysed the terminology for affect appearing in 134 CERME affect papers (Figure 10.4). We first identified altogether 51 different affect terms appearing in the titles and then searched for and counted all appearances of these words in the texts. After removing false positives, the number of occurrences of affect terms was 17368. The most frequently appearing terms were Belief, Emotion, Affect(ive), Motivation, Goal, Efficacy, and Attitude. The frequencies of the term Attitude fell after CERME 4 while the frequencies of the terms Motivation and Goal increased. The term Emotion was popular in CERME 3 and then again in CERME 8 and 9. New concepts have also been introduced to the group over the years; for example the concept of personal meaning (Vollstedt, 2010) emerged in CERME 6.

Next, we present Hannula and Garcia Moreno-Esteva’s (2017) results of a network analysis of affect papers in CERME 4 to CERME 10. The analysis used graph theory to identify groups of papers citing the same authors. The results suggest ten groups: Foundation (30 papers), Self-Efficacy (11 papers), Motivation (11 papers), Teacher Development (8 papers), Resilience (5 papers), Academic Emotions (4 papers), Metacognition (4 papers), Meaning (4 papers), and Identity (4 papers), Teacher Beliefs (3 papers). As many of the groups are quite small a more detailed examination is useful.

[Insert Figure 10.4 here]

Figure 10.4: Frequency of affect term appearance in CERME affect TWG research reports as percentage of all affect terms used

The largest group, Foundation, was united by citing some key researchers in the field of mathematics related affect (e.g. McLeod, Schoenfeld, and Goldin) and some active CERME affect group participants (Hannula, Zan, Pehkonen, and Di Martino). Most of these papers used theoretical frameworks where affect (7 papers), attitude (8 papers), belief (7 papers), or emotion (10 papers) were among the key concepts. Rather than identifying a separate research tradition, this group represents the common ground largely shared by CERME affect papers. Looking at the other groups identified, we see that belief research appears in three groups: Foundation, Self-Efficacy, and Teacher Beliefs. Likewise, emotion appears in Foundation and Academic Emotions.

Taken together, the results confirm the results of the previous analysis of affect terms in articles, identifying affect, belief, attitude, motivation, and emotion as the key concepts. Moreover, both analyses suggest that research on self-efficacy beliefs is somewhat separated from other belief research. Results support the distinction between cognition (beliefs), motivation, and emotions as proposed by Op ‘t Eynde (Figure 10.2) and Hannula (Figure 10.3) models. In the highly influential McLeod (1992) framework, motivation was addressed as a subset of beliefs (motivational beliefs) and may explain why motivation was a rare term until CERME 5. However, since then motivation has become an established key concept for studying mathematics affect, Philippou and Pantziara being most frequent contributors. Among the several theoretical approaches that have been developed in the realm of educational psychology the most influential in CERME has been the achievement goal theory (Elliot, 1999).

The separate group on Identity advocates the relevance of the distinction between psychological and sociological theories. The first papers that discussed identity (Kaasila, Hannula, Laine, & Pehkonen, 2006; Gómez-Chacón, 2006) were classified in the Foundation group, suggesting their need to relate to the dominating frameworks of that time, which has no longer been necessary in more recent CERME papers.

Finally, the analysis suggests dynamics of change as a possible additional characterising feature for research, exemplified by the groups Teacher Development and Resilience. This might add another category between the fluctuating state aspect and more stable trait aspect in the temporal dimension of the Hannula model (Figure 10.3).

2. Key findings of the CERME affect research

In order to discuss further some key findings of the studies presented in affect meetings, we chose to concentrate on studies that involve students, since chapters 12 and 13 of this book specifically consider teachers’ knowledge, beliefs, identity and development. The main focus of such studies over the years concerned (1) the structure of affect and the relation between affective variables and achievement, (2) the role of affect in mathematical problem solving and problem posing, and (3) change in students' affect. Some results in these areas are presented below.

2.1 The structure of affect and the relation between the different affective variables and achievement

In this focus area, the results of the different studies concentrate on the most frequently appearing terms described earlier (beliefs, attitudes, emotions and motivation).

The results of many studies in the TWG with various age-groups of students supported the belief structure suggested by McLeod in 1992 (which refers to beliefs about mathematics, about self, about mathematics teaching and about the social context), and also revealed important patterns regarding the relation between beliefs and other affective variables. Studies in the group also confirmed the role of social variables like the school context and gender in the formation of different beliefs. Particularly, in different CERME meetings, researchers (Gagatsis, Panaoura, Deliyianni & Elia, 2010; Kapetanas & Zachariades, 2007; Rösken, Hannula, Pehkonen, Kaasila, & Laine, 2007) reported the development and properties of scales meant to investigate students’ beliefs in mathematics and mathematics learning/teaching. Some scales were more extended than others, including various dimensions. A closer look these scales revealed factors like beliefs about mathematics and mathematics learning, beliefs related to the personal aspect (self-efficacy, emotional expression, competence, effort) as well as the social aspect, the role of the teacher and the family encouragement.

The results show that positive beliefs and self-efficacy beliefs are correlated with high performance in mathematics (Gagatsis et al., 2010; Kapetanas & Zachariades, 2007; Nicolaidou & Philippou, 2004). The direction of this relationship, and specifically the reciprocal relationship between 8th-9th grade students’ mathematics self-efficacy beliefs and their performance, was investigated by Sørlie Street, Malmberg and Stylianides (2017) in a longitudinal study.

Studies also showed that the social context was an influential factor in students’ beliefs. The study by Kapetanas and Zachariades (2007) traced differences in 10th-12th grade students’ beliefs related to the type of school (public, private, and technical). The findings by Rösken et al. (2007) included significant differences in the beliefs subscales related to 11th grade students of general or advanced courses with students in advanced courses having more positive beliefs. Hannula (2010) revealed that 11th graders of the same class tended to have similar effort, enjoyment of mathematics, and evaluation of the teacher, while their mathematical confidence was influenced by gender. Panaoura, Deliyianni, Gagatsis & Elia (2011) showed differences in students’ beliefs in respect to school grade (5th-8th grades).

The structure of attitudes has received considerable attention in the discussions of the TWG. Di Martino (2010) investigated the structure of attitudes which emerged from the essays of 1600 students across grades 1-13. Three dimensions of the attitude construct emerged. These are the emotional disposition (concisely expressed by “I like/do not like maths”), an affective one (expressed by “to like” and “to adore”) and one correlated with the idea of success in mathematics (expressed by “to understand” and “clever”). Nicolaidou and Philippou (2004) reported positive correlation between primary students’ attitudes and mathematics achievement.

Research on mathematics-related emotions has become more intense in more recent CERME meetings (Figure 10.4). A main source for the studies was Pekrun's framework of academic emotions (Pekrun, Frenzel, Goetz & Perry, 2007). The framework provides a three dimensional taxonomy of academic emotions referring to activity vs. outcome emotions, to their valence (positive vs. negative emotions) and to the degree of activation (activated vs. deactivated). The framework assumes that control and value appraisals relating to learning, teaching, and achievement are of great importance for students’ and teachers’ emotions. Studies about emotions focussed on investigating different emotions at the same time (Martínez-Sierra, 2015), in investigating the intensity of emotions, factors that develop certain emotions like teaching and types of tasks (Schukajlow, 2015), performance (Pantziara, Pitta-Pantazi & Philippou, 2007) and emotion regulation strategies (Op ’t Eynde, De Corte & Mercken, 2007).

Regarding motivation, some main directions were observed in the research papers. Pantziara and Philippou (2010; 2011), using achievement goal theory, investigated primary students’ different motivational goals (mastery, performance and performance-avoidance) and found that mastery goals are related to positive affective variables (self-efficacy, interest) and behaviour (achievement). Another direction referred to intrinsic and extrinsic motivation (Wæge, 2007). Motivation in terms of needs and goals was investigated qualitatively by Wæge (2010). Tuohilampi (2011) combined self-beliefs and motivational theories (achievement goals) to investigate the discrepancy between real and ideal self. In the realm of motivation, the absence of studies related to values was stressed in the CERME 9 and 10 meetings.

2.2 The role of affect in mathematical problem solving and problem posing

One important area of research on mathematics related affect is the role of different affective constructs in the process of mathematical thinking with problem solving. Some quantitative research reports in the TWG investigated the relationship between multiple affective variables and problem solving and posing. Nicolaou and Philippou (2007) found in their study with students of grade 5 and 6 that students’ perceived efficacy was a stronger predictor of their problem posing ability and their general mathematics achievement than their attitudes.

Studies using qualitative methods described students’ emotions during problem-solving. The analysis of emotional states revealed their significant role in students’ difficulties in problem solving, emphasising the relationship between affective states and cognitive aspects. Antognazza, Di Martino, Pellandini and Sbaragli (2015) discussed that the distinction between students’ positive or negative emotions in a specific activity derived from an assessment of the difficulty of the activity proposed (intrinsic aspects), or from more general aspects (e.g. I do not like mathematics). Liljedahl (2017) studied students’ problem solving behaviour when faced with an imbalance between their skills and the challenge of the task. Results indicate that most students have perseverance in the face of challenge and tolerance in the face of the mundane and they use these as a buffer in order to autonomously correct the imbalance between skill and challenge that they experience.

Assuming the existence of a mutual influence of affective and cognitive factors, a case study by Furinghetti and Morselli (2006) revealed that among the elements that shaped the behaviour of a good problem solver were aesthetic values and feelings of freedom in facing the problem. In the same vein, Viitala (2015) described a grade 9 high achiever’s mathematical thinking through problem solving and mathematics related affect. The results revealed a successful, though quite unsure, problem solver whose affective state (connected to problem solving) seemed to tell the same story as her affective trait (view of mathematics). The differences between results on affective state and trait seemed to be connected mostly to emotions.

2.3 Change in students’ affect

The analysis of the papers suggested dynamics of change as a possible additional characterising feature for research. In CERME 6, the affect group identified and reported four different aspects of stability (Hannula, Pantziara, Wæge, & Schlöglmann, 2010): 1) The state and trait aspects of affect; 2) Resistance to change; 3) Robustness of constructs; and 4) Relative stability in relation to other persons.

Studies investigating changes in affect involving students, referred to changes in students’ affect as they move to upper school grades, or the importance of the classroom microculture on students’ affect. Change in students' affect was also observed through their engagement in structured interventions like problem solving and modelling cases or through specific instructional practices.

Past research evidently indicates that students’ mathematics-related affect develops detrimentally during school years. A decline in students’ positive affect was documented by the studies of Athanasiou and Philippou (2010) during the transition to secondary school. Tuohilampi, Näveri and Laine (2015), trying to prevent this decline in students’ affect, applied a three-year intervention designed to improve primary school students’ problem solving skills, and their mathematics-related affect. The impact was restricted but crucial: girls’ affect regarding mathematics decreased less in the intervention group.

Dropping out of mathematics and especially advanced mathematics has become a major concern for society (Moscucci, Piccione, Rinaldi & Simoni, 2006). Moreover, several reports indicate a low rate of tertiary students around the world that are enrolled in science, technology, engineering and mathematics (STEM) related careers with an even lower rate for women (Sánchez Aguilar, Romo Vázquez, Rosas Mendoza, Molina Zavaleta, & Castañeda 2013). Moscucci et al. (2006), highlighting the role of affect in this situation, showed that students who dropped out of school had failed in mathematics the same school year. Other studies have investigated factors that influence students’ enrolment in advanced mathematical courses. Factors referred to students’ perceived competence in mathematics, their future expectations but also to their teachers and relatives (Kleanthous & Williams; 2011; Sánchez Aguilar et al., 2013).

Several studies reported positive change in students’ affect after structured interventions through problem solving (Marcou & Lerman, 2007; Stylianides & Stylianides, 2011) and modelling (Schukajlow & Krug 2013). Barnes (2015) reported on an intervention that explored perseverance in mathematical reasoning in children aged 10–11. The findings suggest improved perseverance because of the effect the intervention seemed to have on the bidirectional interplay between affect and cognition.

Students’ affect is influenced by the learning context and the teacher (Liljedahl & Hannula, 2016). However, the experiences of students in one class may differ and the development of their affect may follow very different paths. Some studies (e.g.Vankúš, 2007) showed increase in students’ affect after implementing some new practices like didactical games and humour in the mathematics classroom. Other studies showed that students’ motivation for learning mathematics can be influenced and altered by changes in teaching (Pantziara & Philippou, 2010; Wæge, 2010).

3. The development of methodology in the field of affect

Methodological issues have always been central in the discussions of the Affect TWG in CERME. Since the first meeting of the thematic working group there have been contributions focused on methodological issues: not only concerning the development of new methods, but also related to theoretical aspects (Evans, 2004). In particular, it has been underlined that researchers’ choices about methodology are related to their affective traits (Zan & Di Martino, 2004) and the choices can condition and constrain the findings (Pantziara, Wæge, Di Martino & Rösken-Winter, 2013).

The reflections about methodology developed over the years in CERME have had a great impact on the development of several critical issues in the field of affect. In the first meetings of the TWG, attention was focused on the definition and delimitation of affective traits. The literature showed the lack of a generally accepted conceptualisation of the principle affective constructs (attitudes, beliefs, emotions, motivation): the different constructs tended to be defined implicitly and a posteriori through the instruments used to measure them (Furinghetti & Pehkonen, 2002; Di Martino & Zan, 2001) and there was no clear distinction between these constructs. Schlöglmann (2004) underlined that the research methods used in the field could not establish a distinction between the categories above and he used this argument to encourage the development of new research methods and approaches. In particular, he suggested considering methods developed in other domains, such as neuroscience.

A second aspect concerns the nature of affective constructs: that it is difficult to infer them. There are essentially two schools of thought: one sees affective constructs as an inner awareness or process of interpretation of events rather than an overt behaviour, and consequently they are not directly observable and, moreover, individuals themselves are often not conscious of these processes (Panaoura & Philippou, 2006). Another school of thought sees affective constructs (such as attitudes and beliefs) not as a quality of an individual but rather as a researcher’s model for describing and understanding some mathematical behaviour (Zan & Di Martino, 2004). In both views, the problems connected with the methodology are evident.

Another important issue affecting the discussion about methodology is the distinction between rapidly changing states and relatively stable traits (Hannula, 2011). Observing affective states seems particularly complicated and there is an imbalance in favour of studies that focus on traits over studies that focus on states. Only in the more recent CERMEs have some studies focused on affective states, and some specific theories (e.g. reversal theory) been introduced as interpretative frameworks (Lewis, 2015; Antognazza et al., 2015). Schlöglmann (2004) argued that quantitative methods reveal stable and less intense categories, while qualitative methods are able to grasp quickly changing and very intense reactions. Actually, the first qualitative studies presented in our CERME group were typically case studies (e.g. Furinghetti & Morselli, 2006) or small sample studies (Liljedahl, Rolka & Rösken, 2007) developed to observe changes in action.

New issues and new goals in the research on affect have been identified in the last 20 years. We can highlight two main directions: the first one follows the traditional approach in the field of affect, searching for causal relationship between affective variables and mathematical performance or behaviour. In this frame, the crucial action is to measure, privileging quantitative methods. It demands isolating, clearly identifying, and measuring variables in order to interpret statistical results. A necessary part of the studies conducted within this frame is the development of means for the efficient measurement of affective constructs, but also of mathematical performance. The second one – following the interpretive paradigm in the social sciences – abandons the goals of explaining behaviour through measurements and of determining general rules based on a cause-effect model to describe the interaction between affective and cognitive constructs in mathematics education, and focuses on trying to make sense of the observed phenomena from the perspective of participants. This implies a significant shift in focus and, in particular, a movement towards the use of qualitative approaches (Evans, Hannula, Philippou & Zan, 2004).

An evident consequence of these considerations is the trend towards a balancing between the use of quantitative and qualitative methods in the research on affect overcoming the initial preponderance of quantitative methods (Table 1). Since the early TWGs, mixed methods or a hybrid approach (i.e. collecting qualitative data, and analysing them with systematic categorisation and basic statistical analysis, for example, cross-tabulations) have become more popular. Moreover, the shift in focus from the description of a phenomenon to the interpretation of the same phenomenon intensifies the attention on how the collected data are interpreted (Di Martino et al., 2015).

[Insert Table 10.1 here]

Table 10.1: The proportions of different methodological approaches in the CERME affect TWG

Concerning quantitative methods, the development of measurement tools is particularly critical. The majority of the quantitative papers have used classic and consolidated questionnaires and scales, but two interesting trends emerge: the first one, coherent with the consideration of the complexity of the affective factors, is the trend to modify and combine two or more scales for the same studies (Pantziara and Philippou, 2011); the second is the trend to adopt the more complex computational tools that have become available to analyse the data (Mosvold, Fauskanger, Bjuland & Jakobsen, 2011).

Concerning qualitative approaches, CERME papers have often introduced new methods to collect data. An exemplary case is offered by Kaasila, Hannula, Laine and Pehkonen’s paper (2006): it discusses the potential of autobiographical narratives to reconstruct the mathematical identity of a person. The key assumption is that humans are storytelling beings who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world. Such new approaches and observational or other tools were developed for specific goals and specific contexts. Perhaps, precisely their specificity is a point of weakness, because it is very difficult to use these instruments when some conditions change.

4. Affect and Mathematical Thinking Group - Looking ahead

The research on mathematics-related affect has repeatedly raised the terminological issues as a problem (e.g. Furinghetti & Pehkonen, 2002; Hannula, 2011; 2012). This problem is related to the cumulative nature of the research, and therefore to the need that new research builds on a critical analysis of previous research. The CERME group has been and will be an important place to highlight this fundamental issue and to tackle it. Yet, it is important to keep a way open for new concepts to emerge. It seems clear, in retrospect, that motivation and identity were terms that were necessary for the research field. It also seems reasonable that we need specific terms, for example, for “Perseverance” (Barnes, 2015) and “Resilience” (Lee & Johnston-Wilder, 2011).

Despite the improved understanding of mathematics related affect, the general trend still is that enjoyment of mathematics decreases over the school years. We need to develop teaching approaches that promote a positive relationship with mathematics without compromising understanding of concepts. Such approaches should be tested through systematic longitudinal intervention studies.

There are three specific methodological possibilities that can open yet newer understandings of the dynamics of mathematics-related affect. The first possibility would be to analyse the dynamics of group level processes: How does the teacher initiate and maintain excitement and good working climate in the class? What kind of processes lead to the ‘energy’ of the class being lost? Concepts like classroom climate would be useful concepts for this kind of analysis. The other new methodological possibility is to implement physiological measures (e.g. heart rate monitoring) to gain a continuous indication of participants’ affective states. These methods have been used for a long time in laboratories, but have only recently become more affordable and usable in actual classrooms. The third methodological possibility is strongly associated with the CERME spirit of collaboration. Comparative research on mathematics-related affect has confirmed that while some research findings about affect are universal, some other findings are contextual. Therefore, we should examine which results about affect are transferable to different sociocultural contexts, and Europe with its diverse educational systems and linguistic groups is a wonderful test bed for such comparative studies.

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