On the non-compliance of a polluting mine under an emission tax ∗†

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Abstract

The production of different exhaustible resources, and mining in particular, are a source of stock pollutants and the growth of these stocks can be constrained by abatement and taxation. However, the firm may have incentives to non-comply with these regulations. This study analyzes the enforcement of an emission tax in a dynamic model, which includes stock externalities and abatement by the mining firm and by natural processes. The purpose of this investigation is two-fold. First, to study the consequences of a given enforcement scheme in a dynamic model and to generalize the results from previous static models. Second, to analyze the time paths of control variables and the violation level. It is argued that non-compliance in itself does not necessarily imply that net emissions from the mine are excessive compared to the full-compliance emission level. It is the possibility of a zero emission report that may cause the divergence from the socially optimal emission levels. The time path of violation and hence the path of unpaid taxes can have multiple shapes depending on the type of tax applied.

Keywords: Emission tax; Enforcement; Exhaustible resources; Monitoring; Mining; Stock pollution.

JEL codes: Q30, Q38, Q50, Q58.

1 Introduction

The mining industry has a large impact on the environment due to the release of pollutants into ecosystems, the production of large quantities of waste rock and tailings, and the loss of habitats. The mining industries are highly regulated in many countries due to these environmental impacts. Nevertheless, these industries are often criticized for their


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excessive environmental impacts and the regulators for their inability to monitor and enforce environmental policies related to mining. Pollution and enforcement problems have been noted in public media such as newspapers in the United States and Finland:

“Data available to EPA shows that, in many parts of the country, the level of significant non-compliance with permitting requirements is unacceptably high and the level of enforcement activity is unacceptably low.”

“Mining in Finland is governed by EU pollution laws but conservationists warn that supervision and control of the industry is poor and government has often failed to monitor or act because the industry and the authorities are closely and intimately linked.”

In Australia, which is one of the world’s leading mining countries, mining regulation and its enforcement have also received criticism from the public agencies and academics:

“We found serious weaknesses in the monitoring of compliance with environmental conditions. As a result, we cannot give assurance that agencies are adequately aware of non-compliance or if environmental conditions are delivering the desired outcomes.”

“In the apparently rare event that disciplinary action is taken against an offending industrial facility, the penalties issued for the relevant breaches of the air pollution laws or standards are usually insufficient to deter recidivism.”

The enforcement problems seem to be a global issue, and their analysis and solution is relevant for the design of effective environmental policy regarding mining. When designing the pollution control instrument the regulator must also decide how to monitor

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1. Environmental Protection Agency (2009). This memorandum is available in Duhigg (2009).
5. Even though problems exist, mining companies do get convicted of environmental crimes. A recent example is the case of the Talvivaara Sotkamo mining company in Finland. The company gave inaccurate information in their environmental permit application and was found guilty of environmental degradation. The total fines and compensation to the state amounted to 3.8 million euros. (Yle News 2016)
and enforce the policy, and to do this, the regulator must keep track of the mining firm’s incentives about whether or not to comply with the regulation. In addition, as non-compliance in the mining industry is not uncommon in practice, it is important to examine the effects of possible non-compliance. The purpose of this study is to investigate mining firm’s compliance decision and its consequences, when it is regulated with an emission tax together with a weak enforcement scheme which allows for non-compliance. The model combines a polluting exhaustible resource producer model with an enforcement model, and contributes therefore into two strands of literature.

Related literature. The study is related to the literature on polluting exhaustible resource production and environmental policy. There is a relatively small body of literature that analyzes environmental policy in the mining sector. For example, Roan and Martin (1996) studied the regulation of a polluting mine with an emission standard and Cairns (2004) applied Roan’s and Martin’s model to green accounting and added capacity constraint and investment to the model. Farzin (1996) studied the optimal taxation of a polluting mine that is causing stock and flow externalities, but he omitted the natural cleaning processes from his stylized model. Farzin’s focus was on the production stage of the mine. Sullivan and Amacher (2009) analyzed the problem of mine land restoration and the optimality of the used policy. The model in this study assumes that firm is regulated by an emission tax. Tahvonen (1997), Ulph and Ulph (1994) and Hoel and Kverndokk (1996) found that the time path of the optimal emission tax has an inverted U-shaped form. In a mining context, Lappi and Ollikainen (2018) analyzed the optimal emission taxes for a mine with two different pollution stocks and found similar results. None of these analyzes incorporate monitoring and enforcement or possible non-compliance into the analysis.

Contrary to the literature on mining and environmental policy, the literature on the monitoring and enforcement of environmental policy instruments is large, but little interest has been dedicated to the polluting exhaustible resources, environmental policy instruments and compliance. Instead, the literature has focused on general properties of the enforcement of environmental policy (Harford, 1978; 1987; Malik, 1992; Sandmo, 2002), pollution permits (Malik, 1990; Stranlund and Dhanda, 1999; Stranlund and Chávez,
2000; Lappi, 2016), individual transferable quotas in fisheries (Hatcher, 2005), technological development (Arguedas, Camacho, and Zofío, 2010) and prices versus quantities-discussion (Montero, 2002; Stranlund and Moffitt, 2014). Dynamic models include Harrington (1988) and Livernois and McKenna (1999) on pollution standards, Montero (2002) on emission tax and emissions trading, Stranlund, Costello, and Chávez (2005) and Lappi (2017) on emissions trading and Arguedas, Cabo, and Martín-Herrán (2017) on optimal emissions standards and non-compliance. Although dynamic models exist in the literature, there are no studies that consider exhaustible resources, emission tax and non-compliance. The present study is therefore to the author’s knowledge the first to analyze the monitoring and enforcement of an emission tax applied to polluting exhaustible resource producer.

**Results and contribution.** The main aim is to analyze the incentives of a polluting exhaustible resource firm, whose pollution is controlled by a time-dependent tax instrument and who must self-report its emissions to the regulator. The truthfulness of these reports is verified through audits, and if a firm is found non-compliant, he must pay a fine that depends only on the level of violation and not explicitly on time. As emphasized above, there are only handful of dynamic models found in the monitoring and enforcement literature, and dynamic model is the suitable description for an exhaustible resource producer. Thus the first goal is to ascertain which results from the static models carry over to the dynamic model. For example, what is the necessary and sufficient condition for compliance? Given the assumption that the expected penalty is a strictly increasing and strictly convex function of the violation level, are the optimal values of the control variables the same as under full-compliance as in the multiple static models? It is shown that the condition for compliance is similar to those found in the static models: that is, a firm is compliant precisely when the marginal expected penalty at zero violation is greater than or equal to the ongoing tax rate. Regarding the second question, the optimal choice of the firm matches the choice under full-compliance when the firm reports positive emissions during the planning or operation interval. These results are generalizations of the results from the static models. However, in Section 4.2 a very rough sufficient, but not necessary, condition is derived for zero emission report (that is, for a report that states
the firm abates all of its emissions). In a dynamic model, the firm may report positive emissions at some initial interval and zero emissions later on. When this happens, the full-compliance emission levels are not guaranteed.

It is not possible to analyze the evolution of the control variables and the violation level of the firm over time using a static model. As one may expect, the violation level has a close relationship with the emission tax, which is the marginal benefit of non-compliance, and therefore it is important to define the types of taxes used in the analysis. The literature, particularly Tahvonen (1997), has shown that the optimal emission tax has an inverted $U$-shaped form using an infinite horizon model without abatement option by the decision maker. Since the operation period of a mining firm is finite, the current study assumes three different types of taxes in the analysis: strictly increasing, inverted $U$-shaped and a constant tax. Since the condition for compliance at any instant of time involves the current tax rate, a compliant firm regulated by an strictly increasing tax can clearly turn non-compliant at some time instant and stay non-compliant for the rest of the planning interval. If a non-compliant firm is regulated by a tax with an inverted $U$-shaped time path, it can turn compliant at a later date given that the emission tax decreases enough. How then does the level of violation evolve over time?

To answer this question, the marginal expected penalty must be less than the tax at some instant of time so that the mining firm begins to under-report its emissions. It is shown, that when the mining firm is regulated with an increasing emission tax, the firm decreases its emission report during the production stage of the mine. If the emission report remains positive during the planning interval, then extraction is strictly decreasing, but abatement and violation are strictly increasing for a non-compliant firm. It is shown that in this case the actual emissions are at the same level as under full-compliance. The intuition for an increasing violation level is straightforward, because the violation profile will be increasing exactly when the marginal benefit of non-compliance (the tax rate) is increasing. It is possible that the emission report becomes zero at some point in time. When this occurs the extraction path is decreasing during the production stage but abatement and violation paths have inverted $U$-shapes.

The violation level is also analyzed in the other two cases. In the first case the emission
tax has an *inverted U-shaped profile* and it is found that the violation level has a time path which at first increases and then decreases. The maximum violation occurs either during the instant of time with maximum tax rate or before it. The second one is a *constant emission tax*. This kind of tax induces either a strictly decreasing violation profile or a profile in which the violation level is first constant and then strictly decreasing.

**Policy implications.** First, a mining firm that is compliant initially can turn non-compliant later on depending on the details of the enforcement policy. Hence a given enforcement policy may not result in compliance in the future even though it has produced full-compliance in the past. But the results show that if the regulator has applied an inverted U-shaped emission tax, then the firm may even turn compliant at a later date. Second, although this does not necessary lead to a more deteriorated environment compared to the full-compliance scenario, it does imply a decrease in the collected amount of taxes. For example, if the taxes are used to finance the rehabilitation of the mine site, the non-compliance may imply that there are insufficient funds for rehabilitation when the mine has been shut down. Currently the collected rehabilitation funds are often short of true rehabilitation costs, which means that non-compliance may make this problem even worse if the funds from the tax are used for rehabilitation. In addition, non-compliance and zero reported net emissions from the mine may imply that the pollution stock, and hence the pollution damage, is greater compared to full-compliance. A weak enforcement policy that induces non-compliance may result in an excessively polluted mine environment unless the emission report stays positive.

This study continues in the next section to develop the mining model and the main assumptions. Section 3 contains a characterization of the allocation under full-compliance whereas the firm is allowed to be non-compliant in Section 4. The last section concludes the study by discussing policy implications and further work that may be needed.

\[\text{(Lappi (2017)) reached this same conclusion in a dynamic model related to emissions trading and non-compliance.}\]
2 The model

The mining firm faces a given time-dependent tax $\Gamma(t)$, which is used by the regulator to control the flow of net emissions from the mine. Before presenting the model’s notation, it is important to note that this model takes the emission tax and the terminal date of the operation as given and focuses on the consequences of non-compliance. The planning interval is fixed to $[0, T]$, where $T$ may be given by the environmental permit of the mine. In this sense the regulation of the mining firm consists of an emission tax and a fixed terminal time. When making its compliance decision, the firm takes the regulation as given including the length of the planning interval.

Net emissions at time $t$ is the difference between the emissions and the abatement. Emissions at time $t$ are $\epsilon q(t)$, where $\epsilon$ is a constant emission factor and $q(t)$ is the rate of extraction. Abatement at time $t$ is denoted by $a(t)$. Hence the quantity of taxes the firm must pay at time $t$ is $\Gamma(t) \left( \epsilon q(t) - a(t) \right)$. The resource stock is exhaustible and its size at time $t$ is denoted with $X(t)$. The initial stock of the resource is $X(0) = x_0$ and the rate of change in the resource stock satisfies the following equation

$$\dot{X}(t) = -q(t).$$

(1)

The resource stock at the terminal time is constrained to be non-negative, that is $X(T) \geq 0$. The price of the resource is a fixed constant $p$. Constant price has been assumed for example by Caputo (1990), Roan and Martin (1996), Cairns (2001) and Cairns (2004).

Extraction of the resource is costly, and for simplicity it is assumed that this cost is captured by $C(q)$, in which the function $C$ satisfies properties $C' > 0$ and $C'' > 0$. The abatement cost is a function of abatement at time $t$ and this function is denoted with $A$. Abatement cost function satisfies the properties $A' > 0$ for $a > 0$ and $A'' > 0$. In addition, the following assumption on price and costs is imposed to obtain the results:

**Assumption 1.**
(i) \( p - C'(0) > 0; \)

(ii) \( A'(0) = 0. \)

Part (i) indicates that the resource price is greater than the marginal cost at zero production level, and Part (ii) states that the cost of the first abatement unit is approximately zero. The assumption that abatement cost is approximately zero at zero abatement level, allows to show that the abatement level is strictly positive during the production stage of the mine. The interest rate is denoted with \( \rho \) and the firm is assumed to maximize the total discounted profit over the planning interval.

Section 4.3 analyzes the effects of non-compliance on the pollution stock. To do that, the pollution stock dynamics must be described. The pollution stock at time \( t \) is denoted by \( N(t) \). It is assumed that the time development of the pollution stock is given by

\[
\dot{N}(t) = \epsilon q(t) - a(t) - \delta N(t),
\]

with an initial value \( N(0) = 0 \). Hence, the pollution stock is increasing in the net emissions. The term \(-\delta N(t)\) describes the depreciation of the pollution stock through the natural processes (exogenous decay). This form for the dynamics is used for example in Tahvonen (1997), Hoel and Kverndokk (1996) and in Ulph and Ulph (1994).

3 A brief characterization of the allocation under full-compliance

In this section the allocation of extraction and abatement under full-compliance is characterized since in the next section the emission level under non-compliance is compared to the full-compliance emission level. Given \( \Gamma(t) \) and \( T \), the objective of the mining firm is to maximize the total discounted profits from mining subject to constraints, that is, to

\[
\max_{\{q(t),a(t)\}} \int_{0}^{T} e^{-\rho t} \left[ pq(t) - C(q(t)) - A(a(t)) - \Gamma(t) \left( \epsilon q(t) - a(t) \right) \right] dt \quad \text{(3)}
\]

s.t. \( \dot{X}(t) = -q(t), \quad X(0) = x_0, \quad X(T) \geq 0, \quad q(t) \geq 0, \quad a(t) \geq 0. \quad \text{(4)} \)

The current value Hamiltonian related to this problem is

\[
H(q, a, X, \lambda) = pq - C(q) - A(a) - \Gamma(\epsilon q - a) - \lambda q, \quad \text{(6)}
\]
where $\lambda$ is the shadow value of the resource on the ground. The Hamiltonian measures
the instantaneous net profit that is augmented by the imputed costs. The necessary
conditions obtained by applying the Maximum Principle can be written as

$$H_q = p - C'(q) - \epsilon \Gamma - \lambda \leq 0, \quad q \geq 0, \quad qH_q = 0, \quad (7)$$

$$H_a = -A'(a) + \Gamma \leq 0, \quad a \geq 0, \quad aH_a = 0, \quad (8)$$

$$\dot{X} = -q, \quad (9)$$

$$\dot{\lambda} = \rho \lambda, \quad (10)$$

$$\lambda(T) \geq 0, \quad X(T) \geq 0, \quad \lambda(T)X(T) = 0. \quad (11)$$

These conditions will be used in Proposition 4.

4 Non-compliance

Suppose that the mining firm must give a report of its net emissions, $\hat{e}$, to the regulator
and pays the tax bill according to that report. However, that report may present an
underestimate of the actual net emissions, in which case the firm is non-compliant. The
regulator may audit the firm and thus the extent of the possible non-compliance may
be revealed. If the mining firm is found guilty of non-compliance, that is, deliberately
reporting net emissions that are smaller than the actual emissions, it must pay a penalty.
It is assumed that the level of the penalty depends on the level of the violation. The
violation level at time $t$, $v(t)$, is defined by equation

$$v(t) = \epsilon q(t) - a(t) - \hat{e}(t). \quad (12)$$

The expected penalty function is denoted by $F$ and it is assumed that it satisfies the prop-
erties $F' > 0$ and $F'' > 0$. This function is defined as the product of the auditing prob-
ability and the penalty function; both of which may depend on the level of violation as is as-
sumed in numerous static settings such as Harford (1987), van Egteren and Weber (1996),
Also some of the dynamic models, such as Stranlund, Costello, and Chávez (2005),
Arguedas, Cabo, and Martín-Herrán (2017) and Lappi (2017), assume that the expected

penalty depends on the violation level. With price-based instruments the monetary penalty set by the regulator is often a constant multiplied by the violation level (Stranlund, 2007; Stranlund and Moffitt, 2014).

Again, the objective of the mining firm is to maximize the total discounted profits from mining subject to constraints, but now the firm pays the tax based on the emission report and is faced with an additional constraint, \( v(t) \geq 0 \), which together with the non-negativity constraints defines the control set. The violation level is assumed to be non-negative, since the firm has no incentives to over-report its emissions. The problem for the mining firm is then to

\[
\max_{\{q(t), a(t), \hat{e}(t)\}} \int_{0}^{T} e^{-\rho t} \left[ pq - C(q(t)) - A(a(t)) - F(v(t)) - \Gamma(\hat{e}(t)) \right] \, dt 
\]  

s.t. \( \dot{X}(t) = -q(t) \), \( X(0) = x_0 \), \( X(T) \geq 0 \),  

\( \varepsilon q(t) - a(t) - \hat{e}(t) \geq 0 \),  

\( q(t) \geq 0 \), \( a(t) \geq 0 \), \( \hat{e}(t) \geq 0 \).  

Neglecting for now the non-negativity constraints in (16), the generalized Hamiltonian is

\[
L(q, a, \hat{e}, X, \lambda, \eta) = H(q, a, \hat{e}, X, \lambda) + \eta(\varepsilon q - a - \hat{e}),  
\]

where \( H \) is the current value Hamiltonian given as

\[
H(\cdot) = pq - C(q) - A(a) - F(\varepsilon q - a - \hat{e}) - \Gamma \hat{e} - \lambda q,  
\]

and \( \eta \) is a multiplier that depends on time. Taking into account the non-negativity constraints, the necessary conditions given by the Maximum Principle can be written
\[ L_q = p - C'(q) - F'(v)\epsilon - \lambda + \eta\epsilon \leq 0, \quad q \geq 0, \quad qL_q = 0, \quad (19) \]
\[ L_a = -A'(a) + F'(v) - \eta \leq 0, \quad a \geq 0, \quad aL_a = 0, \quad (20) \]
\[ L_{\hat{e}} = F'(v) - \Gamma - \eta \leq 0, \quad \hat{e} \geq 0, \quad \hat{e}L_{\hat{e}} = 0, \quad (21) \]
\[ \dot{X} = -q, \quad (22) \]
\[ \dot{\lambda} = \rho\lambda, \quad (23) \]
\[ \lambda(T) \geq 0, \quad X(T) \geq 0, \quad \lambda(T)X(T) = 0, \quad (24) \]
\[ \eta \geq 0, \quad v \geq 0, \quad \eta v = 0. \quad (25) \]

To interpret these conditions, suppose that the controls are strictly positive. Then by (21), the emission report is chosen such that \( \Gamma = F'(v) - \eta \). Expression (19) implies then that a positive extraction level is chosen to equalize the resource price with the full marginal cost, which includes: the marginal cost of production, the marginal emission tax and the shadow value of the resource stock. Expression (20) implies that a positive abatement level is chosen to equalize the marginal abatement cost with the emission tax, which is the marginal benefit of abatement. Note that, when the firm is non-compliant, the multiplier \( \eta \) is zero. In this case a positive emission report is chosen to equalize the marginal expected penalty with the going emission tax; that is, the marginal cost of non-compliance is equated with the marginal benefit of non-compliance.

The following three subsections analyze these conditions. Section 4.1 contains results that are independent of the time path of the tax as the derived results hold for any of the analyzed emission taxes and include generalizations of the results in the previous studies that use static monitoring and enforcement models. Sections 4.2 and 4.3 contain results, which depend on the qualitative properties of the time path of the tax, and which can be obtained only by applying a dynamic model.

\[ ^{11}\text{Theorem 2 of Chapter 2 in Seierstad and Sydsæter (1987). It is assumed that the solution consists of } C^3 \text{ or even } C^2\text{-functions without explicit mention. It is shown in Lemma 1 that the optimal controls are continuous functions.} \]
4.1 A characterization of the non-compliance decision and its consequences

The aim of this section is first to analyze the firm’s compliance decision. After that the effect of the abatement technology level, resource price and other parameters on the compliance choice of the firm is analyzed. Finally, a sufficient condition to obtain the emission level under full-compliance is derived. The following proposition is useful for obtaining some of the main results of this study.

Proposition 1.

(i) The controls are continuous functions of time.

(ii) The extraction rate is positive on some interval.

(iii) If the rate of abatement is positive at some instant of time, the rate of extraction is also positive at that same instant of time.

(iv) A non-compliant firm never chooses zero abatement.

Proof. See Appendix A.1.

Therefore by Part (iv), a non-compliant mining firm chooses abatement such that the marginal abatement cost equals the marginal expected penalty (or the marginal benefit of abatement). Note, however, the key role of assumption $A'(0) = 0$ in obtaining this result. Without this assumption, a sufficient condition for the result is $F'(0) > A'(0) > 0$. That is, if the marginal expected penalty at zero violation is greater than the marginal abatement cost at zero abatement, the non-compliant firm abates some of the emissions.

The first main result characterizes the compliance choice of the firm. As expected from the static results of the literature, whether the firm chooses to comply or not, depends on the ordering of the marginal expected penalties and the going tax rate.

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12The results in this section hold also when the resource price is allowed to be time dependent given that Part (i) of Assumption 1 is changed into $p(t) - C'(0) > 0$ for all $t \in [0, T]$. In the next section, where the time paths of the controls are investigated, it is unclear how time dependent resource price would affect the results.
Proposition 2. Suppose that net emissions, $\epsilon q(t) - a(t)$, are positive. Then the following characterization of compliance holds:

$$F'(0) \geq \Gamma(t) \iff v(t) = 0. \quad (26)$$

Proof. Suppose first that $v = 0$. Then $\dot{e} > 0$ and $F'(0) - \Gamma - \eta = 0$. This equation implies $F'(0) \geq \Gamma$, because $\eta \geq 0$. Suppose then that $F'(0) \geq \Gamma$. Assume on the contrary to the claim that $v > 0$. Then $\eta = 0$ and $F'(v) \leq \Gamma$. Thus $F'(v) \leq \Gamma \leq F'(0)$, implying by assumption $F'' > 0$ that $v \leq 0$. This contradicts $v > 0$. □

It is assumed in Proposition 2 that the net emissions are positive. Note that when the net emissions are zero, the firm is automatically compliant by the constraint $v \geq 0$. The result means that the choice of the penalty function (given the auditing probability) and its relation to the tax induces the firm to be either compliant or non-compliant. Given the auditing probability, the regulator can induce compliance using a penalty function, whose marginal expected value at zero violation level equals the tax rate for every instant of time. This result mirrors that found in numerous studies in static settings that include Malik (1990), Malik (1992) and Stranlund and Dhanda (1999), and in the dynamic setting in Lappi (2017). In addition, the result remains valid even though here non-compliance is investigated in an exhaustible resource production model.

The result in Proposition 2 can be combined with the different types of taxes considered here. To this end, suppose that enforcement is weak in the sense that the firm begins to non-comply at some instant of time. When the firm is regulated with an increasing emission tax, the firm will continue to non-comply until the terminal time when the mine is closed, since the tax will exceed the marginal expected penalty at zero violation level. But with the inverted $U$-shaped tax, a non-compliant firm may begin to comply with the regulation at a later time instant, if the tax falls by a sufficient amount.

It is well-known from the static monitoring and enforcement literature, in particular from the study by Stranlund and Dhanda (1999), that the changes in the level of abatement technology have no effect on the level of violation (including the choice between compliance and non-compliance), and that it only affects the choice of emissions. The current model is a dynamic model, in which the net emissions are defined as
To investigate how an exogenous shift in abatement costs influences the level of violation, it is supposed that the abatement costs depend on parameter $\theta$ describing the used technology. The abatement costs are now redefined as $A(a; \theta)$ with $A_{ab} < 0$. Thus, as $\theta$ increases (technology improves) the marginal abatement costs decrease, as in Stranlund and Dhanda (1999).

**Proposition 3.**

(i) The instant of time, in which the mining firm turns from compliant to non-compliant (or vice versa), is independent on the level of the abatement technology used by the firm.

(ii) An improvement in the abatement technology used by the mining firm has no impact on the violation level, when the firm reports positive emissions.

(iii) An increase in the resource price, emission factor or in the interest rate has no impact on the violation level, when the firm reports positive emissions. In addition, the instant of time, when the firm turns non-compliant is not affected by a change in either of these parameters.

**Proof.**

(i) The instant of time, in which the firm turns from compliant to non-compliant (or the other way around) is defined by the solution to equation $F'(0) = \Gamma(t)$. Neither $F$ nor $\Gamma$ depend on the state of technology $\theta$, which implies that the instant of time in question is independent of $\theta$.

(ii) When the firm is non-compliant with positive emission report, equation $F'(v) - \Gamma = 0$ holds and it readily follows from this equation that $v_\theta = 0$.

(iii) The proof is similar to the proof of parts (i) and (ii).

This result shows that the time instant at which a firm turns compliant or non-compliant is independent of the level of the abatement technology and this outcome generalizes the static result reported by Stranlund and Dhanda (1999). The second generalization shows that a change in the abatement technology has no effect on the violation level given that the firm reports positive emissions. However, in the current model, but

$eq - a^{13}$ In Stranlund and Dhanda (1999) the choice variables are the emissions and the amount of permits. Here the choice is made on the amount of extraction and abatement, which define the net emissions.
not in the static model of \textit{Stranlund and Dhanda (1999)}, it is possible that the firm reports zero emissions at some point in time (see the beginning of the Subsection \ref{sec:4.2} for a sufficient condition for zero emission report). In this case the effect on the violation level becomes ambiguous, when the tax is such that \( F'(v) - \Gamma < 0 \) holds.\footnote{In the next section an increasing emission tax is analyzed and this inequality holds, if the firm reports zero emissions, because the violation is decreasing after reported emissions hit zero (see Proposition \ref{prop:5}).} The most interesting contents of Part (iii) shows that the mining firm’s compliance decision is independent of changes in the level of the resource price or in the interest rate.

The final result of this section is important, because it states that given a positive emission report during the mine’s production stage the state of the environment is the same as with the extraction and abatement levels under full-compliance. This result is similar to those found in many static monitoring and enforcement settings such as those reported by \textit{Harford (1978), Malik (1990), Stranlund and Chávez (2000) and Sandmo (2002)}.

**Proposition 4.** Suppose \( \hat{e} > 0 \) on the whole planning interval. Then the optimal values of the other controls (\( q \) and \( a \)) and also the state variables (\( X \) and \( N \)) match the values under full-compliance.

\textit{Proof.} Since \( \hat{e} > 0, \ F'(v) - \eta = \Gamma \) by condition (21). Inserting \( F'(v) - \eta = \Gamma \) into the first inequalities of conditions (19) and (20) yields the following inequalities

\begin{align*}
p - C'(q) - \epsilon \Gamma - \lambda & \leq 0, \quad (27) \\
-A'(a) + \Gamma & \leq 0. \quad (28)
\end{align*}

These conditions together with the corresponding complementary slackness conditions and conditions (22)–(24) match conditions (7)–(11).

When the reported emissions stay strictly positive throughout the planning interval, the mining firm will choose the extraction and abatement rates to equal the rates under full-compliance. In this case the values of the state variables (\( X \) and \( N \)) also match the full-compliance values. This means that the environmental goal will be met, when the mining firm always reports positive emissions. If the regulator only cares about the state of the environment, there is no reason to worry about non-compliance unless the firm
reports that no net emissions are produced by the mine. Note that since the mining firm’s production technology allows for abatement the report of zero emissions at some instant of time may simply mean that the firm abates all of its emissions.

However, a non-compliant mining firm decreases the amount of taxes paid and a regulator who must gather money or is budget constrained may also need to audit a mining firm reporting positive emissions. This result has some further policy implications. From the environmental point of view, it is not necessary for the regulator to design the enforcement scheme to guarantee compliance during to whole production stage. Instead it is enough to set a scheme that induces non-zero emission report.

4.2 The time path of the violation level and the control variables

The results obtained thus far are independent of the qualitative properties of the tax rate. The main emphasis of this section is the time path of the violation level. As will be shown, the form of this time path depends on the chosen emissions tax. The three different emission taxes to be studied are an increasing emission tax, an inverted U-shaped tax and a constant tax. It is assumed that the taxes are continuously differentiable with respect to time. The time path of the violation is interesting because it gives information on the unpaid taxes. Whenever the violation level increases over time, the difference between the actual net emissions and the emission report will increase by the definition of the violation level. Multiplying this difference by the going tax rate, one obtains the amount of unpaid taxes. Hence, the information on the time path of the violation is simultaneously also information on the unpaid taxes, and therefore a relevant aspect to study. The results in Proposition imply that the amount of unpaid taxes is independent of the state of technology, resource price, emission factor and interest rate given that the firm reports positive emissions.

It is shown in the following, that the time path of the emission report and whether or not the report obtains a zero value at some point in time is key to the analysis of the time path of the violation level. For this reason, and also for the result in Proposition it is relevant to know under which conditions the emission report can be zero at optimum. A very rough sufficient, but not necessary, condition for zero emission report
is \( \Gamma > F'(\epsilon q^\text{max} \max) \), where \( q^\text{max} \) solves equation \( p - C'(q) = 0 \). In other words, for a sufficiently high emission tax the optimal emission report is zero.

For most of the results in this section it is assumed that the shadow value of the resource stock \( \lambda \) is non-zero. Note that this assumption excludes the possibility that \( X(T) > 0 \). At the end of the section, the possibility that \( X(T) > 0 \) is briefly analyzed.

**Increasing emission tax.** The analysis begins with an increasing emission tax satisfying \( \Gamma'(t) > 0 \) for all \( t \in [0, T] \). It is assumed that the firm turns non-compliant at some instant of time \( t_1 \), implying that the tax satisfies inequalities

\[
\begin{cases}
\Gamma(t) < F'(0), & t \in [0, t_1), \\
\Gamma(t) > F'(0), & t \in (t_1, T],
\end{cases}
\]

and equation \( \Gamma(t_1) = F'(0) \). It is therefore assumed that the enforcement is weak. This condition indicates that the firm begins to non-comply at time instant \( t_1 \) and continues to do so for the rest of the production stage as described by Proposition 2, because the tax is strictly increasing. The time paths of the control variables and violation are derived by first considering the interval \((t_1, T)\) and then the interval \([0, t_1)\). It turns out that the violation and abatement levels are increasing if the emission report stays positive. But if the report becomes zero, then the violation and abatement levels begin to decrease. It is also shown that the extraction is strictly decreasing over time.

These results are derived by first presenting a series of lemmas concerning the time derivatives of the controls and corner solutions of the emission report (in the sense that it is optimal to choose a zero emission report).

**Lemma 1.**

(i) \( \dot{\epsilon} = 0 \) on an interval \( I \subset (t_1, T] \) implies \( \dot{q} < 0, \dot{a} < 0 \) and \( \dot{v} < 0 \) on \( I \),

(ii) \( \dot{\epsilon} > 0 \) on an interval \( I \subset (t_1, T] \) implies \( \dot{q} < 0, \dot{a} > 0, \dot{\epsilon} < 0 \) and \( \dot{v} > 0 \) on \( I \).

**Proof.** See Appendix A.2. \( \square \)

Because \( \dot{a} = F''\epsilon \dot{q}/(A'' + F'') \) on the interval where \( \dot{\epsilon} = 0 \) (see the proof in Appendix A.2) and because \( F''/(A'' + F'') \in (0, 1) \), the rate at which emissions \( \epsilon q \) decrease is

\( \dot{\epsilon} = 0 \) by (21).
higher than the rate at which abatement decreases, which implies that the violation level decreases. From this lemma it is evident that the qualitative results regarding violation level depend on the behavior of the optimal emission report, and specifically whether or not there are corner solutions. The following two lemmas concern these kinds of solutions.

**Lemma 2.** The emission report is strictly positive on some interval $[t_1, t_+)$, where $t_+ \leq T$.

*Proof.* See Appendix A.3.

**Lemma 3.** If $\dot{e} = 0$ at some instant of time $t_0 > t_1$, then $\dot{e} = 0$ for the rest of the interval $(t_0, T]$.

*Proof.* See Appendix A.4.

Lemma 2 states that as the firm begins to non-comply, it will still choose to report a strictly positive emission level to the regulator at least for some time after the violation level begins to increase. Lemma 3 indicates that if the firm chooses to report zero emission level, it will continue to do so until the end of the planning interval. The time paths of the optimal controls are next characterized on the interval $(t_1, T]$, in which the firm is non-compliant. Let $t_0 \in (t_1, T]$ be the instant of time at which the emission report becomes zero (if such an instant exists).

**Proposition 5.** Consider the interval $(t_1, T]$. The time paths of extraction, abatement, emission report and violation are characterized as follows:

(i) extraction is strictly decreasing;

(ii) emission report is strictly decreasing when $\dot{e} > 0$;

(iii) abatement is strictly increasing on the interval $(t_1, t_0)$ and strictly decreasing on the interval $(t_0, T]$;

(iv) violation is strictly increasing on the interval $(t_1, t_0)$ and strictly decreasing on the interval $(t_0, T]$. 
Proof. Parts (i) and (ii) follow directly from Lemma 1. Parts (iii) and (iv) also follow from Lemma 1 after noting that by lemmas 2 and 3 $\dot{e} > 0$ before $t_0$ and $\dot{e} = 0$ after $t_0$.

An interpretation for these results is given after the full picture of the time paths is obtained. To study the time paths on the interval $[0, t_1]$, in which the firm is compliant, it is first shown that the controls are strictly positive there.

Lemma 4. Extraction, abatement and emission report are all strictly positive on the interval where the firm is compliant, that is, on $[0, t_1]$.

Proof. See Appendix A.5

The next proposition characterizes the extraction, abatement and emission report on the interval $[0, t_1]$ and completes the analysis of the case with an increasing emission tax.

Proposition 6. Consider the interval $[0, t_1]$. The time paths of extraction, abatement and emission report are characterized as follows:

(i) extraction is strictly decreasing;

(ii) abatement is strictly increasing;

(iii) emission report is strictly decreasing.

Proof. (i) $\dot{q} < 0$ as in (A.24) of the proof of Lemma 1.

(ii) $\dot{a} > 0$ as in (A.25) of the proof of Lemma 1.

(iii) Because $\dot{v} = 0$ on $[0, t_1]$, one obtains $\dot{e} = \epsilon \dot{q} - \dot{a} < 0$ by Parts (i) and (ii).

Figure 1 illustrates the results in propositions 5 and 6. The instant of time $t_1$ satisfying equation $F'(0) = \Gamma(t)$ divides the mine’s production stage into two parts: In the first part, the firm is compliant and in the second part the firm is non-compliant. Results show, that during the mine’s production stage the extraction rate will decrease and the abatement rate will increase over time (the latter holds, if the emission report is positive). During the first part, where the firm is compliant, the emission report decreases and continues to decrease even after the time instant in which the firm becomes non-compliant.
Figure 1: Illustration of the time paths of extraction and emission report (left), abatement (middle) and violation level (right), when an increasing emission tax is used as the emission control instrument. Here $\hat{e}_{t_0}$, $a_{t_0}$ and $v_{t_0}$ are the optimal emission report, abatement and violation, when the emission report obtains zero value at the instant of time $t_0$.

By choosing extraction, abatement and emission report the firm effectively chooses the violation level, and after the time instant $t_1$, but before $\hat{e}$ hits zero, the firm chooses the level of violation such that equation $F'(v) = \Gamma$ holds. That is, the violation level is chosen to equalize the marginal expected penalty with the going emission tax, or in other words, the marginal cost of non-compliance is equated with the marginal benefit of non-compliance. This rule holds to the point, in which the firm starts to report zero emissions. At this time instant the violation level obtains its maximum level and afterwards the level of violation starts to decrease. Hence the lost tax revenue due to non-compliance is either strictly increasing or has an inverted U-shaped time path, when the firm is regulated with an increasing emission tax and turns non-compliant at some point during the production stage.

The optimal abatement satisfies equation $A'(a) = F'(v)$ when the firm under-reports its emissions. Hence, the marginal abatement costs are equalized with the marginal benefit of abatement, which is the marginal expected penalty for under-reporting emissions. This equation shows that the time paths of abatement and violation level are qualitatively the same.

Inverted U-shaped emission tax. The time path of violation is studied next, when an inverted U-shaped tax is used to control the emissions. Define $t_{\text{max}}$ as the instant of time at which the tax rate obtains its maximum value. It is assumed that $\Gamma'(t) > 0$ for

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16To analyze fully the time paths of the controls in this case seems to be a difficult thing to do for the author and is omitted.
all $t \in [0, t_{\text{max}})$ and $\Gamma'(t) < 0$ for all $t \in (t_{\text{max}}, T]$. Furthermore, suppose that the firm turns non-compliant at some time instant $t_1$ and compliant at another instant $t_2$. Then the tax satisfies inequalities

$$\begin{cases}
\Gamma(t) < F'(0), & t \in [0, t_1), \\
\Gamma(t) > F'(0), & t \in (t_1, t_2), \\
\Gamma(t) < F'(0), & t \in (t_2, T],
\end{cases}$$

and equations $\Gamma(t_1) = F'(0)$ and $\Gamma(t_2) = F'(0)$. Note that $t_{\text{max}} \in (t_1, t_2)$. See Figure 2 for an illustration.

**Proposition 7.** Consider the interval $(t_1, t_2)$ in which the firm is non-compliant. The time path of violation first increases and then decreases with the maximum violation occurring either at $t_{\text{max}}$ or at some $t_0 \in (t_1, t_{\text{max}})$.

**Proof.** See Appendix A.6

Figure 2 contains the two possibilities for the time path of violation. The figure on the left illustrates the compliance choice and the middle figure the time path of the violation for a positive emission report. In this case the maximal violation occurs at the instant of time, in which the emission tax (the marginal benefit of non-compliance) is the highest, and the shape of the violation level time path follows the time path of the tax. The figure on the right contains the other possibility for the violation, in which the emission report becomes zero at some point in time when the tax is still increasing. As with an increasing emission tax, the equation $A'(a) = F'(v)$ also holds in this case. This implies that the time path of the abatement is qualitatively similar as the time path of the violation level.
Figure 2: Illustration of the time path of violation when a tax with an inverted $U$-shaped time path is used as the emission control instrument. The figure on the left contains an illustration of the choice of compliance mode and the time instants at which it changes. The other two figures illustrate the two possibilities for the time path of the violation level and its maximum value.

**Constant emission tax.** The previous results have characterized among other things the level of violation when the firm is regulated with an increasing tax or with a tax with an inverted $U$-shaped time path. In practice, however, the chosen tax level may be constant. Intuition suggests that a constant emission tax induces the firm to choose constant abatement so that the marginal abatement cost is equated with the constant tax. This intuition rests on the assumption of full-compliance. To investigate this, suppose that a constant emission tax is used to control pollution. Suppose also that the tax is set at a higher value than the marginal expected penalty (at zero violation level) implying that the firm is always non-compliant. Again, the main result is developed using lemmas.

**Lemma 5.** Suppose that the emission tax is constant and $\Gamma > F'(0)$. If $\hat{e} = 0$ at some instant of time $t_0 \geq 0$, then $\hat{e} = 0$ for the rest of the interval $[t_0, T]$.

*Proof.* See Appendix A.7.

**Lemma 6.** Suppose that the emission tax is constant and $\Gamma > F'(0)$. If $\hat{e} > 0$ at some instant of time $t_1 \geq 0$, then $\hat{e} > 0$ on some interval $[t_1, t']$. In addition, if $t_1 > 0$, then $\hat{e} > 0$ on some interval $[0, t']$.

*Proof.* See Appendix A.8.

Lemma 5 states that as the reported emissions become zero, they stay zero, and Lemma 6 indicates that the emission report is strictly positive during some interval after the initial time, if it is strictly positive somewhere. Define $I_+ = [0, t_0)$ as the interval in
which \( \hat{e} > 0 \). This interval can be empty, that is, it is possible that the emission report is zero on the whole production stage, but when \( I_+ \) is non-empty, it divides \([0, T]\) into two parts, one with \( \hat{e} > 0 \) and other with \( \hat{e} = 0 \), by lemmas 5 and 6. The following result characterizes the optimal paths.

**Proposition 8.** Suppose that the emission tax is constant and \( \Gamma > \Gamma'(0) \). The time paths of extraction, emission report, abatement and violation are characterized as follows:

(i) extraction, abatement and violation are strictly decreasing on \([0, T] \setminus I_+;\)

(ii) extraction and emission report are strictly decreasing on \( I_+ \), and abatement and violation are constant on \( I_+ \).

**Proof.** See Appendix A.9.

Figure 3 illustrates this result.

Note that the following equations characterize the violation and abatement choices of the firm, when \( \hat{e} > 0 \):

\[
F'(v) = \Gamma \quad \text{and} \quad \Gamma = A'(a).
\] (31)

The reason for constant abatement and constant violation, is therefore that these constant values equate the marginal cost of non-compliance with the constant marginal benefit of non-compliance, and the constant marginal benefit of abatement with the marginal cost of abatement. However, after the time instant \( t_0 \) at which the emission report becomes zero, the violation level is \( v = \epsilon q - a \). Since the extraction decreases over time, the
abatement and the violation levels can no longer stay at the previous constant levels. Instead, the abatement level is chosen such that it satisfies the equation

\[ A'(a) = F'(e q - a), \tag{32} \]

which states that the marginal abatement cost equals the marginal benefit of abatement. The marginal benefit of abatement is decreasing in abatement, and as time goes on, the extraction decreases, which shifts the marginal benefit curve inwards. Hence the optimal abatement decreases.

To end the investigation of the time path of violation, consider the case in which some of the resource is left in the ground. When this is optimal for the firm, the shadow value of the resource is zero. This seems to complicate the analysis, when the emission report can obtain value zero. But when the report is strictly positive, equation \( F'(v) - \Gamma = 0 \) holds. Then \( \dot{v} = \dot{\Gamma} / F''(v) \). Therefore the time path of the violation level is qualitatively similar to the time path of the tax. In particular, an inverted U-shaped tax implies an inverted U-shaped time path for the violation level.

### 4.3 Additional results: The time path of the pollution stock

This section derives some additional results related to the time path of the pollution stock under full-compliance and under non-compliance. Proposition 4 states that if the firm reports positive emissions during the whole production interval, then the actual net emissions are exactly at the full-compliance level, which implies that the pollution stock follows the full-compliance pollution stock. However, the firm may report zero emissions effectively stating that there are no net emissions from the mine due to abatement efforts. This may change the properties of the pollution stock time path.

Consider first the pollution stock under the full-compliance scenario. Tahvonen (1997) has shown in his model that the pollution stock has an inverted U-shaped time path for a low initial pollution stock value. As already stated, the main differences between the current model and Tahvonen’s model are related to the terminal time and abatement options. These, in particular abatement by the mining firm, complicate the analysis regarding the pollution stock. Consider first full-compliance and the model in Section 3.

**Proposition 9.** Suppose that the firm complies with the regulation.
(i) If the tax has either an increasing or a constant time path, then the pollution stock has an inverted U-shaped time path or is strictly increasing.

(ii) If the tax has an inverted U-shaped time path, $\dot{q} < 0$ and the mine owner cannot abate, then the pollution stock has an inverted U-shaped time path or is strictly increasing.

Proof. See Appendix A.10.

When the mining firm is non-compliant with the regulation at some point in time, a similar result about the pollution stock is obtained as for the full-compliance case.

Proposition 10. Suppose that the firm is non-compliant at some point in time.

(i) If the tax has either a strictly increasing or constant time path, then the pollution stock has an inverted U-shaped time path or is strictly increasing.

(ii) If the tax has an inverted U-shaped time path, $\dot{q} < 0$ and the mine owner cannot abate, then the pollution stock has an inverted U-shaped time path or is strictly increasing.

Proof. See Appendix A.11.

It is important to note that the non-compliance case and the full-compliance case are only qualitatively similar. Thus the results do not imply that the pollution stocks are necessarily the same, since the firm may report zero emissions. It is possible that in the non-compliance case the pollution stock obtains a larger value by the terminal time than under full-compliance. This indicates that the damages from the pollution stock will be larger compared to the full-compliance case.

5 Conclusions and discussion

This paper analyzed a mining model, in which the polluting mine is regulated by an emission tax and may be non-compliant during the production interval. The mine emits a stock pollutant and is regulated by an emission tax that may depend on time. The mining firm is required to self-report its net emissions to the regulator and pay the tax bill
according to that report. However, the quantities of emissions in the report may be less than the actual emissions from the mine. If so, the regulator may notice this, and in that case the regulator sanctions the firm based on the size of the violation. As the previous literature has studied the optimal emission tax on an exhaustible resource producer, the purpose of this paper is to extend the literature on polluting exhaustible resources by adding the enforcement of the emission tax and the possibility of non-compliance to the model. Three types of taxes were considered: a strictly increasing tax, an inverted $U$-shaped tax and a constant tax.

The first set of results holds for all the applied tax types and generalize the static model results. They indicate that multiple enforcement properties, such as the condition for (non-)compliance and the possibility for full-compliance emission level, carry over from the static models to the dynamic model. For example, positive emission report implies the same emission level as under full-compliance. However, the emission report in a dynamic model can be positive on one interval and zero on another, and a positive emission report at the beginning and zero emission report later on, may imply that the full-compliance emissions are not achieved.

In the second set of results the time path of the tax set by the regulator plays a central role. Here, the emphasis is on the time paths of extraction, abatement, emission report and, in particular, violation. When the mining firm is enforced by a weak scheme that allows for non-compliance, the time path of the violation may increase until the end of the production interval (increasing tax) or may even peak at some instant and decrease thereafter (inverted $U$-shaped tax; also possible with an increasing tax). With a constant tax, the violation level is constant unless the firm begins to report zero emissions, which implies that the violation level begins to decrease.

These results have some policy implications. First, a mining firm that is compliant initially can turn non-compliant later on depending on the details of the enforcement policy and on the applied tax scheme. Hence a given enforcement policy may not result in compliance in the future even though it has produced full-compliance in the past. Moreover, a non-compliant firm may turn compliant at a later date if it is regulated by an inverted $U$-shaped tax.
Second, although non-compliance does not necessarily lead to a more deteriorated environment compared to full-compliance, it does imply a decrease in the collected amount of taxes. Therefore, if the sole purpose of setting an emission tax on the mining firm’s emissions is to constrain the emissions to their socially optimal levels, non-compliance with the instrument may not be a problem. However, if there is a need to collect revenue, non-compliance with the emission tax will be a problem. For example, an emission tax for a mine may be supported by the need to collect revenue for the rehabilitation of the mine site. Non-compliance may endanger sufficient fund raising for this purpose.

Third, non-compliance combined with zero reported net emissions from the mine may imply that the pollution stock, and hence the pollution damage, is greater compared to full-compliance. A weak enforcement policy may result in an excessively polluted mine environment unless the emission report is positive throughout the production lifetime of the mine.

A few words about model’s assumptions and possible extensions are in order. A key assumption, although very typical in the relevant monitoring and enforcement literature, is that the expected penalty depends on the absolute size of the violation. An alternative for this is to assume that the expected penalty depends on the relative size of the violation, that is, on the ratio between the violation size and the emission report as in Hatcher (2005, 2012) and Lappi (2016). This assumption may correspond more closely with practice, but it also seems to result in significantly more complex analysis in particular regarding the time path of the violation level. In addition, it is reasonable to expect that the expected penalty depends on the level of past non-compliance, since the enforcer may target auditing at firms who were found non-compliant in the past. Therefore one interesting possibility for further research would be to assume fixed compliance periods during the production stage where possible auditing occurs at the end of every compliance period and penalties that depend on the verified non-compliance of the past periods. Furthermore, the penalty may depend also on the "seriousness of the offense" (Billiet, Blondiau, and Rousseau, 2014), which may be interpreted as a case where the penalty depends on the size of the pollution stock at the time of the violation.

In the countries mentioned in the introduction the applied regulatory instrument
is an emission standard, which is also used in Canada and Sweden (Söderholm et al., 2015) along with the other member states of the European Union often as a part of the environmental permit for the mine. The main extension to the current model is to analyze the monitoring and enforcement of an emission standard and to compare the results with those found here using an emission tax. Furthermore, an analysis on mine land rehabilitation, instrument choice, and monitoring and enforcement is clearly an important avenue for future work both from the theoretical and policy points of view. The optimal design of the rehabilitation mechanism should take into account the cost of rehabilitation and, in addition, the effort done by the firm and the final result, since according to Mudd (2010) monitoring of past rehabilitation works has shown that rehabilitation does not always work well and the pollution problems may persist after rehabilitation.
A Appendix

A.1 Proof of Proposition 1

(i) Controls are continuous because the control set is
\[
\{(q, a, \hat{e}) \in \mathbb{R}^3 \mid q \geq 0, \ a \geq 0, \ \hat{e} \geq 0, \ \epsilon q - a - \hat{e} \geq 0\}
\] (A.1)
is convex and \(H\) is strictly concave in controls.

(ii) It is shown that \(q(t) > 0\) at least for one time instant, which implies the claim by the continuity of \(q\). Suppose on the contrary to the claim that \(q \equiv 0\). Since \(q \equiv 0\), \(x(T) = x_0 > 0\), and therefore \(\lambda \equiv 0\). Since \(q \equiv 0\), it must hold by the constraint \(v \geq 0\) that \(v \equiv 0\) and \(a \equiv 0\). Condition (19) then becomes
\[
p - C'(0) - F'(0)\epsilon + \eta \epsilon \leq 0.
\] (A.2)
From (20) and from assumption \(A'(0) = 0\) (recall Assumption 1), \(F'(0) - \eta \leq 0\). This and assumption \(p - C'(0) > 0\) imply
\[
p - C'(0) - (F'(0) - \eta)\epsilon > 0,
\] (A.3)
contradicting inequality (A.2).

(iii) If \(a > 0\) at some instant of time, also \(q > 0\). Otherwise one would have \(v < 0\).

(iv) When \(v > 0\), \(\eta = 0\). Therefore, the choice of abatement satisfies the inequality
\[-A'(a) + F'(v) \leq 0.\]
Suppose to the contrary of the claim that \(a = 0\) for some time instant. Then assumption \(A'(0) = 0\) implies \(F'(v) \leq 0\), which contradicts \(F'(v) > 0\).

A.2 Proof of Lemma 1

(i) The firm is non-compliant during the interval \((t_1, T]\), that is, \(v > 0\). This implies that \(q > 0\) on the interval \((t_1, T]\) and Part (iv) of Proposition 1 implies that \(a > 0\) on the interval \((t_1, T]\). Suppose that \(\hat{e} = 0\) on an interval \(I \subset (t_1, T]\). The time derivatives \(\dot{q}\) and \(\dot{a}\) are calculated by differentiating equations
\[
p - C'(q) - F'(v)\epsilon - \lambda = 0,
\] (A.4)
\[-A'(a) + F'(v) = 0,
\] (A.5)
with respect to time. This gives
\[[-C'' - F'' \epsilon^2] \dot{q} + F'' \epsilon \dot{a} - \dot{\lambda} = 0,\]  
(A.6) 
\[F'' \epsilon \dot{q} + [-A'' - F''] \dot{a} = 0.\]  
(A.7) 

Equation (A.7) can be written as
\[\dot{a} = \frac{F'' \epsilon \dot{q}}{A'' + F''} \]  
(A.8) 

Plugging this into equation (A.6) and organizing one obtains
\[\dot{q} = \frac{\dot{\lambda}}{-C'' - F'' \epsilon^2 + \frac{F'' F'' \epsilon^2}{A'' + F''}} < 0.\]  
(A.9) 

From (A.8) follows that \(\dot{a} < 0\). These imply \(\dot{v} < 0\) on \(I\), because
\[\dot{v} = \epsilon \dot{q} - \dot{a} = \epsilon \dot{q} \frac{A''}{A'' + F''} < 0.\]  
(A.11) 

(ii) Suppose that \(\hat{e} > 0\) on an interval \(I \subset (t_1, T]\). Differentiating the following equations with respect to time
\[p - C'(q) - F'(v) \epsilon - \lambda = 0,\]  
(A.12) 
\[-A'(a) + F'(v) = 0,\]  
(A.13) 
\[F'(v) - \Gamma = 0,\]  
(A.14) 
one obtains
\[\dot{q} + \frac{\dot{\lambda}}{-C''} < 0,\]  
(A.18) 
\[\dot{a} = \frac{\dot{\Gamma}}{A''} > 0,\]  
(A.19) 
\[\dot{\hat{e}} = \frac{\epsilon \dot{\lambda}}{-C''} - \dot{\Gamma} \left[\frac{\epsilon^2}{C''} + \frac{1}{A''} + \frac{1}{F''}\right] < 0.\]  
(A.20)
Now using the definition of violation level and (A.18)-(A.20) (or directly equation (A.14)) one obtains

\[ \dot{v} = \frac{\dot{\Gamma}}{F''} > 0. \quad (A.21) \]

### A.3 Proof of Lemma 2

It is first shown that there exists an open interval starting at \( t_1 \), on which the firm’s emission report is strictly positive. Suppose on the contrary to the claim that \( \hat{e} = 0 \) on any interval \((t_1, t_1 + \alpha)\) with a (small) \( \alpha > 0 \). Lemma 1 implies \( \dot{v} < 0 \) on \((t_1, t_1 + \alpha)\). But by the continuity of \( v \), this contradicts the facts \( v(t_1) = 0 \) and \( v > 0 \) on \((t_1, t_1 + \alpha)\). Thus \( \hat{e} > 0 \) on some interval \((t_1, t_1 + \alpha)\). It is now shown that \( \hat{e} > 0 \) on the interval \([t_1, t_1 + \alpha]\). To do this, it must be shown that \( \hat{e}(t_1) > 0 \). By Lemma 1 \( \hat{e} < 0 \) on \((t_1, t_1 + \alpha)\). This and the continuity of \( \hat{e} \) imply \( \hat{e}(t_1) > 0 \). Number \( t_+ \) can now be defined for example by \( t_+ = t_1 + \alpha/2 \).

### A.4 Proof of Lemma 3

Suppose on the contrary to the claim that \( \hat{e}(t_2) > 0 \) for some \( t_2 > t_0 \). Then by the continuity of \( \hat{e} \) there must exist an interval \( I \subset (t_1, T) \) such that \( \hat{e} \) is strictly increasing on \( I \). But this contradicts inequality (A.20) in the proof of Lemma 1.

### A.5 Proof of Lemma 4

\( q > 0 \): Note that \( q(t_1) > 0 \), because \( q \) is continuous and strictly decreasing on \((t_1, T]\). Since \( q(t_1) > 0 \) and \( q \) is continuous, \( q \) is strictly positive at least just before \( t_1 \). Suppose that there exist \( t \in [0, t_1) \) such that \( q(t) = 0 \). Since \( q(t_1) > 0 \), there exists a largest \( t \) such that \( q(t) = 0 \). Denote it by \( t_2 \). By the continuity of \( q \), there exists an interval, \( I \subset [t_2, t_1] \), where \( \dot{q} > 0 \). On that interval either \( \hat{e} > 0 \) or \( a > 0 \) (or both are positive) in order \( v = 0 \) to hold. Suppose first that \( \hat{e} > 0 \) and \( a \geq 0 \). Then (21) implies \( F'(0) - \eta = \Gamma \). Plugging this to (19), one obtains after differentiation that

\[ \dot{q} = \frac{1}{C''(q)} [-\dot{\lambda} - \dot{\Gamma} \epsilon] < 0, \quad (A.22) \]
which contradicts $\dot{q} > 0$ on $I$. Suppose then that $a > 0$ and $\dot{e} = 0$. Using (20) and (19), one obtains

$$\dot{q} = \frac{1}{C''(q)}[-\dot{\lambda} - A''(a)\dot{a}], \quad (A.23)$$

which is greater than zero on $I$, implying $\dot{a} < 0$. However, since on $I$, $\dot{q} > 0$, $v = 0$ and $\dot{e} = 0$, the abatement rate must increase on that interval, which contradicts $\dot{a} < 0$. Hence $q > 0$ on $[0, t_1]$.

$a > 0$: Suppose $a = 0$ at some instant of time in $[0, t_1]$. Condition (20) includes then the inequality $F'(0) - \eta \leq 0$ by assumption $A'(0) = 0$. Since $\Gamma > 0$, this inequality implies $F'(0) - \eta - \Gamma < 0$. Thus by (21) $\dot{e} = 0$. But because $q > 0$, $a = 0$ and $\dot{e} = 0$ imply $v > 0$, which is a contradiction with $v = 0$ on $[0, t_1]$. Hence $a > 0$ on $[0, t_1]$.

$\dot{e} > 0$: Suppose that there exist a time instant with $\dot{e} = 0$. Because $\dot{e}$ is continuous and $\dot{e}(t_1) > 0$ by Lemma 2, there must exist an interval $I \subset [0, t_1]$ such that emission report is strictly increasing. Since $\dot{e} > 0$ on $I$, one obtains from equations (19), (20) and (21), that

$$\dot{q} = \frac{1}{C''(q)}[-\dot{\lambda} - \dot{\Gamma}e] < 0, \quad (A.24)$$

$$\dot{a} = \frac{\dot{\Gamma}}{A''} > 0. \quad (A.25)$$

The violation level is constant on $I$ ($v = 0$), implying $\dot{v} = 0$. Using this, (A.24) and (A.25), it follows that $\dot{e} = \epsilon \dot{q} - \dot{a} < 0$ on $I$, which contradicts the fact that the emission report is strictly increasing on $I$.

A.6 Proof of Proposition 7

Suppose first that $\dot{e} > 0$ on $(t_1, t_2)$, and note that $\dot{\Gamma}(t_{\text{max}}) = 0$. Using this and $\dot{v} = \dot{\Gamma}/F''$, one obtains $\dot{v} > 0$ on $(t_1, t_{\text{max}})$ and $\dot{v} < 0$ on $(t_{\text{max}}, t_2)$. Hence the time path of $v$ is inverted U-shaped, and the violation level obtains maximum value at $t_{\text{max}}$.

Suppose then that $\dot{e} = 0$ at some instant of time on $(t_1, t_2)$ and consider first the interval $(t_{\text{max}}, t_2)$. Again, $\dot{e} > 0$ for some $t$ in $(t_{\text{max}}, t_2)$ implies $\dot{v} < 0$ on some interval $I \subset (t_{\text{max}}, t_2)$. If $\dot{e} = 0$ on an open interval in $(t_{\text{max}}, t_2)$,

$$\dot{v} = \epsilon \dot{q} - \dot{a} = \epsilon \dot{q} - A'' F'' < 0. \quad (A.26)$$
(For calculations, see the proof of Lemma 1) Therefore \( \dot{v} < 0 \) for almost every \( t \in (t_{\text{max}}, t_2) \) (the possibility of \( \dot{v} = 0 \) at points \( \hat{t} \) where \( \dot{e}(\hat{t}) = 0 \) and \( \dot{e} > 0 \) for all \( t \in (\hat{t} - \alpha, \hat{t}) \) and for all \( t \in (\hat{t}, \hat{t} + \alpha) \) for some small \( \alpha > 0 \) is not ruled out here). Consider then the interval \((t_1, t_{\text{max}})\), and suppose that \( \dot{e} \) obtains the value zero there, say at \( t_0 \). It is shown that \( \dot{e} \) stays zero after \( t_0 \) at least until \( t_{\text{max}} \). For this, note that \( \dot{e} > 0 \) on any interval \( J \subset (t_1, t_{\text{max}}) \) implies (see the proof of Lemma 1 and recall that \( \dot{\Gamma} > 0 \) on \((t_1, t_{\text{max}}))\)

\[
\dot{e} = \frac{\epsilon \dot{\lambda}}{-C''} - \dot{\Gamma} \left[ \frac{e^2}{C''} + \frac{1}{A''} + \frac{1}{F''} \right] < 0 \quad \text{on } J, \tag{A.27}
\]

and therefore \( \dot{e} \) cannot increase on \((t_1, t_{\text{max}})\). Hence \( \dot{e} = 0 \) for all \((t_0, t_{\text{max}})\). This and

\[
\dot{v} = \epsilon \dot{q} - \dot{a} = \epsilon \dot{q} \frac{A''}{A'' + F''} \tag{A.28}
\]

imply \( \dot{v} < 0 \) on \((t_0, t_{\text{max}})\). On the interval \((t_1, t_0)\), \( \dot{e} > 0 \), which implies \( \dot{v} > 0 \). Hence the time path of \( v \) first increases and then decreases, and the level of violation obtains maximum value at \( t_0 \).

### A.7 Proof of Lemma 5

(The proof is similar as the proof of Lemma 3) Let \( \hat{e}(t_0) = 0 \) and suppose that \( \hat{e}(t_2) > 0 \) for some \( t_2 > t_0 \). Then by the continuity of \( \hat{e} \) there exist an interval \( I \subset [0, T) \) such that \( \hat{e} \) is strictly increasing on \( I \). But this contradicts the following inequality

\[
\dot{e} = \frac{\hat{e} \dot{\lambda}}{-C''} - \hat{\Gamma} \left[ \frac{\hat{e}^2}{C''} + \frac{1}{A''} + \frac{1}{F''} \right] = \frac{\hat{e} \dot{\lambda}}{-C''} < 0, \tag{A.29}
\]

which is calculated as in the proof of Lemma 1.

### A.8 Proof of Lemma 6

The first part follows from the continuity of \( \hat{e} \). For the second part, suppose \( t_1 > 0 \) and \( \hat{e}(t_2) = 0 \) for some \( t_2 \in [0, t_1) \). Then there exists an interval \( I \subset (t_2, t_1) \) such that \( \hat{e} \) is increasing on \( I \). But this also contradicts the inequality

\[
\dot{e} = \frac{\hat{e} \dot{\lambda}}{-C''} < 0. \tag{A.30}
\]
A.9 Proof of Proposition 8

(i) \( \dot{e} \equiv 0 \) on \([0, T] \setminus I_+ \). Therefore extraction, abatement and violation are all strictly decreasing because

\[
\dot{a} = \frac{F'' \dot{q}}{A'' + F''}, \quad (A.31)
\]

\[
\dot{q} = \frac{\dot{\lambda}}{-C'' - F''e^2 \frac{A''}{A'' + F''}} < 0, \quad (A.32)
\]

\[
\dot{v} = \epsilon \dot{q} - \dot{a} = \epsilon \dot{q} \frac{A''}{A'' + F''} < 0, \quad (A.33)
\]

as calculated in the proof of Lemma 1.

(ii) One obtains with similar calculations as those used in the proof of Lemma 1 that

\[
\dot{q} = \frac{\dot{\lambda} + \epsilon \dot{\Gamma}}{-C''} = \frac{\dot{\lambda}}{-C''} < 0, \quad (A.34)
\]

\[
\dot{a} = \frac{\dot{\Gamma}}{A''} = 0, \quad (A.35)
\]

\[
\dot{\epsilon} = \frac{\epsilon \dot{\lambda}}{-C''} - \dot{\Gamma} \left[ \frac{e^2}{A''} + \frac{1}{F''} \right] = \frac{\epsilon \dot{\lambda}}{-C''} < 0, \quad (A.36)
\]

\[
\dot{v} = \frac{\dot{\Gamma}}{F''} = 0, \quad (A.37)
\]

since \( \dot{\Gamma} = 0 \).

A.10 Proof of Proposition 9

(i) The idea of the proof is similar to that described by Tahvonen (1997). First an increasing emission tax is considered. From (7), it is found that the rate of change of extraction satisfies equation

\[
\dot{q} = \frac{1}{C''(q)} [\rho \dot{\lambda} - \dot{\Gamma} \epsilon]. \quad (A.38)
\]

Because \( \dot{\Gamma} > 0 \) for all \( t \), \( \dot{q} < 0 \) for all \( t \).

Suppose that there exists an interval, where the time path of the pollution stock has a U-shape. Since by assumption there exists U-shaped portion, there exists a time instant \( t_1 \) such that \( \dot{N}(t_1) = 0 \), and

\[
\dot{N}(t_1) = \epsilon \dot{q}(t_1) - \dot{a}(t_1) \geq 0 \quad (A.39)
\]

\[17\]Note that this inequality really is of type “\( \geq \)” and not “\( > \)” as Tahvonen wrote.
Because $\dot{\Gamma} > 0$, $\dot{a} > 0$. In particular, $\dot{a}(t_1) > 0$. This and $\epsilon \dot{q}(t_1) - \dot{a}(t_1) \geq 0$, imply $\dot{q}(t_1) > 0$, which is a contradiction with (A.38). Hence there are no $U$-shaped portions on the time path. This implies that the path has an inverted $U$-shaped form or it is strictly increasing. The same proof also applies for the constant emission tax, since extraction is strictly decreasing in that case as is well-known.

(ii) With the restriction $\dot{q} < 0$ and no abatement possibilities, the result follows as in Part (i).

A.11 Proof of Proposition 10

(i) Let the tax be strictly increasing. Propositions 5 and 6 together showed that $\dot{q} < 0$ on $[0, T]$, when the tax is strictly increasing. Define the time instant $t_1$ as in the proof of Proposition 9. Again,

$$\dot{N}(t_1) = \epsilon \dot{q}(t_1) - \dot{a}(t_1) \geq 0. \quad (A.40)$$

There are two possibilities regarding the emission report: either $\hat{e}(t_1) > 0$ or $\hat{e}(t_1) = 0$. When $\hat{e}(t_1) > 0$, $\dot{a}(t_1) > 0$. This and $\dot{q}(t_1) < 0$ imply $\epsilon \dot{q}(t_1) - \dot{a}(t_1) < 0$, which contradicts (A.40). If $\hat{e} = 0$ on an interval containing $t_1$, equation (A.11) in the proof of Lemma 1, that is equation

$$\dot{v} = \epsilon \dot{q} - \dot{a} = \epsilon \dot{q} \frac{A''}{A'' + F''}, \quad (A.41)$$

implies $\epsilon \dot{q}(t_1) - \dot{a}(t_1) < 0$, which contradicts (A.40). If $\hat{e}(t_1) = 0$, but $\hat{e} > 0$ for all $t \in (t_1 - \alpha, t_1)$ for some small $\alpha > 0$, $\dot{a}(t_1) = 0$ and then (A.40) implies $\epsilon \dot{q}(t_1) \geq 0$, which contradicts $\dot{q}(t_1) < 0$. Hence the pollution stock has an inverted $U$-shaped time path or is strictly increasing. The same proof also applies for the constant emission tax, since by Proposition 8 extraction is strictly decreasing.

(ii) With the restriction $\dot{q} < 0$ and no abatement possibilities, the result follows as in Part (i).
References


“Kainuu court reduces charges, fines T Alviaara bosses for environmental crimes.”

http://yle.fi/uutiset/kainuu_court_reduces_charges_fines_talvivaara_bosses_for_environmental_crimes