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No magic: From phenomenology of practice to social ontology of mathematics

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Introduction

In accordance with the maxim ”Back to the things themselves!” the phenomenological approach advocates a return to a close study of mathematical practice to ground philosophical claims. While Mary Leng (2002) has traced the history of phenomenological approach to Lakatos's *Proofs and Refutations*, this paper argues that the view can indeed be based on Husserl's writings. It further claims that by using Husserl's writings, the nature of the phenomenological approach can be developed to obtain a promising philosophy of mathematical practice. However, we will argue that it implies but does not offer, at least not obviously so, a full-fledged metaphysical account on the nature of the objects of mathematics. In this paper, phenomenology of mathematical practice is complemented with social ontological considerations. Our aim is systematic, ultimately motivated by metaphysical rather than phenomenological concerns. We aim to show how to use the phenomenological method for a social ontological approach to mathematics, so that the outcome is a phenomenologically justified account of the ontology of mathematics as practiced. In this view mathematical entities are viewed as abstract and independent of the individual mathematicians' practices (and thus as real), but at the same time “internal” to the socio-historically generated mathematical practice. We aim to do justice to the fact that while the mathematicians investigate the formal realm independent of them, the mathematical discoveries take place in mathematical practice and hence are socially constructed in the rather uncontroversial sense of being products of historically developed mathematical practices.

The first section of the paper is dedicated to arguing that the phenomenological philosophy of mathematics, as outlined by Leng is indeed what Husserl already had in mind and hence the adjective ”phenomenological” is apt to describe it. The second section discusses the metaphysical neutrality of phenomenology. Although the phenomenological method itself is metaphysically neutral, the usage of the method has metaphysical implications. The third section moves to further metaphysical considerations with which we will complement the phenomenological method. The fourth section suggests applying the tools and conceptions from social ontology to mathematics. The social ontological approach we adopt in this paper sees mathematical objects as social constructions, that is, products of the shared mathematical practices (such as proving), and real in a similar way that universities and money and other familiar parts of social reality are real However, mathematical social constructions differ from many other socially constructed entities in that they are highly constrained by various external factors (such as normative constraints, inter-theoretical relations, cognitive abilities, or empirical applicability), and consequently possess a high degree of objectivity. The proposed metaphysical account of mathematical ontology offers a way to flesh out the metaphysical implications of the phenomenological approach, since the account elaborates further both the significance of human practices and historical and social factors, and the experience that mathematics is about abstract things that exist externally to the individual mathematician. The fifth section addresses a possible worry arising from such a view, namely that the existence and creation of socially constructed mathematical objects is merely a piece of metaphysical magic. This 'magic objection' is argued against by showing that we are entitled to treat practice-dependent mathematical objects as legitimately existing things – and not results of any magic tricks – on similar grounds as the social entities we commonly take to exist. The sixth section concludes with an elaboration of how the phenomenological method and the social ontological approach complement each other, and proposes that, together, they form a comprehensive philosophy of mathematical practice.

1. Phenomenological philosophy of mathematics

In her article, "Phenomenology and Mathematical Practice," Mary Leng explained that "[t]he phenomenological philosopher of mathematics starts by taking a good look at mathematics, and only then asks, and tries to answer, philosophical questions about the discipline"(2002, 3). Leng traces the origins of such phenomenological philosophy of mathematics to Lakatos's *Proofs and Refutations*. Her reason to dismiss Husserl is that

> Husserl advocates close consideration of the objects of mathematics, such as numbers, rather than the practices of mathematicians. A phenomenological study of mathematics which followed Husserl's lead would consider our idea of number, for example, and ask how that idea occurs. (Leng 2002, 5)

Instead, Leng proposes using the term 'phenomenological philosophy of mathematics' to describe the practitioner's interest in "the point of view" belonging to mathematics (2002, 5).

Husserl's original focus was, indeed, the concept of number - in his *Philosophy of Arithmetic* (1891), Husserl described our common-sensical idea and origin of the number concept. But, thanks to the more general development in mathematics, Husserl gave up this restriction and already in his *Prolegomena to Pure Logic* (1900),[1] Husserl writes:

> Only if one is ignorant of the modern science of mathematics, particularly of formal mathematics, and measures it by the standards of Euclid or Adam Riese, can one remain stuck in the common prejudice that the essence of mathematics lies in number and quantity. (Husserl 1975/2001, §71)

Husserl thought that formal mathematics is a study of structures, not of number or quantity. With time, Husserl's investigations become more and more encompassing, and sometimes the new findings required revisions in his initial views.[2] By the 1920s he realized that science is an intersubjective praxis and that it does not take place in a vacuum but that its historical development had to be included into its faithful description. This realization applied also to mathematics. Accordingly, in *Formal and Transcendental Logic* (1929), a book that he later held as his most mature publication, Husserl claimed to give a "*definitive clarification of the sense of purely formal mathematics* …, *according to the prevailing intention of mathematicians*: its sense, namely, as a *pure analytics of non-contradiction*, in which the concept of truth remains outside the

[1] The way in which Husserl's view responds to the development in mathematics in the late 19th century is discussed e.g., in N.N. 2010.

[2] He explains the way he made progress and the consequent shift between the first edition of *Logical Investigations* (1901) and the *Ideas I* 1913 with the following words: "as the horizon of my research widened, and as I became better acquainted with the intentional 'modifications' so perplexingly built on one another, with the multiply interlacing structures of consciousness, there came a shift in many of the conceptions formed in my first penetration of the new territory"(2001, 3). His progress to his later "generative" view of phenomenology is similarly a result of painstaking analyses in which he takes more and more factors into account.

theme”(1974, 15-16/1969, 11). Thus he aims to capture what mathematics is according to the prevailing intention of its practitioners in the 1920s.

Husserl then outlines a method to accomplish this. Having first claimed that science is a cultural formation produced ”by the practice of the scientists and generations of scientists who have been building them”(1974, 13/1969, 9), he writes that to understand this practice, philosophers have to enter in *a community of empathy* with the scientists:

> As so produced, they [sciences] have a final sense, toward which the scientists have been continually striving, at which they have been continually aiming. Standing in, or entering, a community of empathy with the scientists [*Einfühlungsgemeinschaft*], we can follow and understand – and carry on ’sense-investigation’ [*Besinnung*]. (1974, 13/1969, 9).

The scientists’ practice is determined by final senses, that is, the goals of the discipline in question that have guided the scientists for generations. Husserl then defines his method as ”sense-investigation” [*Besinnung*] that aims to make explicit scientists’ otherwise typically only vague goals:

> *Sense-investigation* signifies nothing but the attempt actually to produce the sense ”itself,” which, in the mere meaning, is a meant, a presupposed, sense; or equivalently, it is the attempt to convert the ”intentive sense [*intendierenden Sinn*]”…, the sense ”vaguely floating before us” in our unclear aiming, into the fulfilled, the clear, sense, and thus to procure for it the evidence of its clear possibility. (Husserl 1974, 13/1969, 9)

When applied to mathematics, this method aims at explicating the unclear (epistemic) goals of mathematicians. With it, Husserl aims at procuring a practitioner’s point of view by also taking into account the history of the discipline from its inception by the Greeks (for more, see Hartimo 2021). Hence, in *Formal and Transcendental Logic* Husserl first engages in sense-investigation of logic and mathematics, by examining their history as well as the ”living intentions” of the mathematicians. Husserl’s investigation produces a picture of formal logic, which consists of a theory of judgment and formal mathematics, what he calls “*pure analytics of non-contradiction*, in which the concept of truth remains outside the theme”(1974, 15-16/1969, 11). In contrast, applied mathematics (including logic as it relates to the world) aims in addition also at truth, obtained by an encounter of the objects (abstract or real) themselves.

Husserl's method of sense-investigation reveals that the ultimate normative goals of mathematics were epistemic values, such as truth and non-contradiction, but he also gave examples of more concrete goals such as the Euclidean form of a theory (1974/1969, §31). Today one could, of course, identify many more goals and values, such as different kinds of rigor that determine the practice in question.[3] If this kind of method was used today, it would have to consider the subsequent developments and metalogical results, due to which the formal disciplines have fragmented into sub-disciplines with different kinds of more specified goals and epistemic values.

The method of sense-investigation as such is surprisingly close to Maddy's naturalist method as described in her *Naturalism in Mathematics* (1997). Like Maddy's naturalist philosopher, also Husserlian phenomenological philosopher analyzes mathematical aims, points out if these aims are confused or conflict, and examines whether the mathematicians are really reaching their goals.[4] Thus with Leng, we can claim that the phenomenological approach to mathematics, like Maddy's naturalism, strives to understand and evaluate mathematics ”'on its own terms,'” which means abandoning ”the possibility of providing a revisionary philosophy of mathematics for *purely* philosophical reasons” (Leng 2002, 6).

But the phenomenological philosopher differs from Maddy's approach in the phenomenologist's additional recourse to *transcendental* phenomenology. The phenomenologist thinks that the above-described method of sense-investigation should be ”spiced up” with a transcendental clarification of the conditions of possibility of the theoretical research and its results. Without the transcendental clarification, the description of the practice carried out in the straightforward natural attitude is naïve, i.e., non-reflected. Therefore, the phenomenological philosopher as described in *Formal and Transcendental Logic* uses two methods, sense-investigation and transcendental clarification, in combination, to capture the mathematicians' intentions.[5]

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[3] These different norms of rigor could be for example a category theoretical, set theoretical, or type theoretical conception. In lecture notes from 1927 Husserl himself raises an interesting question whether there might be two kinds of ideas of exactness, two different kinds of geometries both supported by the same intuitions, as norms: ”Kann nicht einem und demselben System der Anschauung als einem 'ungefähren' verschiedene Ideen der Exaktheit als Normen untergelegt werden?” (2012, 255).

[4] For more details on how Husserl's phenomenology relates to Maddy's mathematical naturalism, see Hartimo 2020a, 2021.

[5] In his writings in general, Husserl is not restricted to these but he uses all kinds of other methods too, such as the method of free variation, logic, and philosophical thought-experiments (like his discussion of Twin Earth in 1911 (Husserl 1987, 202-219).

In the case of mathematics, the transcendental phenomenological clarification will explicate the presuppositions and the kinds of evidence [*Evidenz*][6] sought for in mathematical practice. The transcendental phenomenological clarification will then help to evaluate which account of mathematical practice is genuine [*echt*], i.e., carried out with clarified concepts, principles, and theories, and so that it fulfills its theoretical goals that are likewise clarified.

Husserl characterizes transcendental clarification as follows:

> In naive intending and doing, the aiming can shift, as it can in a naive repetition of that activity and in any other going back to something previously striven for and attained. […] Turning reflectively from the only themes given straightforwardly (which may become importantly shifted) to the activity constituting them with its aiming and fulfilment – the activity that is hidden (or, as we may also say, 'anonymous') throughout the naive doing and only now becomes a theme in its own right – we examine that activity after the fact. That is to say, we examine the evidence awakened by our reflection, we ask it [the evidence] what it was aiming at and what it acquired; and, in the evidence belonging to a higher level, we identify and fix, or we trace, the possible variations owing to vacillations of theme that had previously gone unnoticed, and distinguish the corresponding aimings and actualizations (Husserl 1974/1969, §69)

Husserl first explains the natural, "naïve" attitude, such as the one in which one is when proving a theorem. After this he explains that the transcendental phenomenological clarification is "turning around" to reflect on this straightforward natural practice. The purpose of such clarification is to point out possible confusions and shiftings in this activity. Transcendental phenomenology (of mathematics) clarifies the kinds of evidence[7] aimed at in the theoretical practice and reveals idealizing presuppositions in mathematics (1974/1969, §§73-81). These are for example, a presupposition of an ideal identity of judgments (§73); reiteration, i.e., that "one can always again"

[6] Note that in writings on phenomenological philosophy, the English term 'evidence,' as a translation for the German *Evidenz*, refers to the seeing or showing that something is so. It should not be understood in the legal sense of the term as what enables someone to see that something is the case, which in German would be *Beweismittel* (A talk given by George Heffernan in Warsaw, in August 17, 2022).

[7] Husserl explains that the goal of non-contradictoriness is given in the evidence of *distinctness* [Deutlichkeit]. Distinctness is a kind of evidence that is acquired from mere judgments, such as, articulate statements and coherent theories. The goal of truth, in turn, is given in the evidence of *clarity* [Klarheit], which is obtained when intuiting what is meant in the judgment, such as the intended state of affairs. These kinds of evidence can be shifted. For example, Hilbert arguably confused them (Hartimo 2021, Chapter 5).

(§74); the logical principles such as the law of contradiction and the law of the excluded middle (§§75-78); *modus ponens* (§78), and the fundamental presupposition that every judgment can be decided (§79). Note that these are described as the practitioners' implicit presuppositions, now explicated, but not postulated as a doctrine.

The transcendental clarification also reveals how abstract objects are given to mathematicians. Husserl, for example, compares the evidence with which the irreal objects, such as the abstract objects of mathematics, are given, to perception. Both kinds of evidence are fallible—"[e]ven an ostensibly apodictic evidence [evidence of necessity] can become disclosed as deception"(§58)—yet all kinds of evidence are of "something-itself," as opposed to a mere picture or some other empty intention of it (such as through a mere sign) (§§58–59). They confront us by something whose being is transcendent (i.e., external to consciousness) (§§60-61). Husserl writes, "[t]he identity and, therefore, the objectness [*Gegenständlichkeit*] of something ideal can be directly 'seen',"(1974/1969, §58). The objects show themselves in evidence more or less perfectly, as related to each other, and as pointing ahead to new ones (§60). Husserl also holds that evidence of physical objects precedes the evidence of abstract (irreal) ones, so that the latter objects refer back to an actual or possible reality (§64).[8]

Ultimately, transcendental phenomenology takes Husserl to the problems of transcendental subjectivity (such as intersubjectivity) to which all sciences are related (§103). This in turn takes the investigator to general phenomenological problems of any kind of experience that can take place in the lifeworld. However, in our systematic interest of understanding *mathematical practice* we will not follow Husserl there, but suggest applying his *method* of transcendental clarification to explore all the kinds of evidence in which mathematical formations are given. It is critical reflection of what has been carried out in formal sciences, and its aim is to spot the possible conceptual confusions and shiftings and to make us aware of idealizing presuppositions.

So, Husserl's phenomenological method is a combination of methods: sense-investigation and transcendental clarification. While the method has critical aspects in aiming at revision of the confused concepts and principles, making sure that they are applied in their proper scopes, and seeking to clarify the kinds of evidence, and so forth, it does not evaluate the subject matter with

[8] Accordingly Husserl's theory of judgments shows how the complex judgments are reducible to elementary judgments about perceived objects. This is explained in detail in Hartimo (2021, Chapter 5; 2020b)

a pregiven external conception of what it should be like. It suggests revisions on the basis of the mathematicians' explicated and clarified intentions "internal" to the practice. Hence, ”the possibility of providing a revisionary philosophy of mathematics for *purely* philosophical reasons” is abandoned, as Mary Leng demands (2002, 6). On present reading, Husserl's transcendental clarification does not even make sense if conceived in an armchair, in isolation from the practices it is about.

2. Phenomenology, its neutrality, and the ontology of mathematics

In *Logical Investigations* Husserl characterizes phenomenology as free from metaphysical, scientific and psychological presuppositions (1984, 28/2001, 179) and for this reason it is thought to be a ”metaphysically neutral” method. David Carr has argued that in this sense, Husserl's enterprise resembles Kant's in being an inquiry into the possibility of metaphysics, mathematics, and science. It does not add to their claims, nor replace them with new claims, but it inquires into how they are possible. Similarly, Husserl's transcendental phenomenology ”does not consist of knowledge claims about the world whether scientific or metaphysical. By 'bracketing' these claims as we have seen, he turns his attention from the world and its objects to the experiences in which they are given. Like Kant, he emphasizes the 'how' question: the '”how” of manners of givenness'” (David Carr 1999, 101). The ”how” of its manner of givenness includes a question of whether the object in question is given as transcendent or not, i.e., its mode of being.[9] As Husserl puts it in *Formal and Transcendental Logic*,

> [e]xperience is the performance in which for me, the experiencer, experienced being 'is there', and is there *as what* it is, with the whole content and the mode of being that experience itself, by the performance going on in its intentionality, attributes it. If what is experienced has the sense of 'transcendent' being, then it is the experiencing that

[9] Obviously, we can be mistaken about whether the transcendent object we are experiencing really is there, for example, if we are experiencing a hallucination. We ultimately take into account the harmonious continuity of our experiences and their agreement with the experiences of others. Our empirical knowledge has an impact as well: we may know that what we are experiencing is a hallucination because we just ingested a drug that produces such experiences, or because it goes against our empirical knowledge about how things are in the world (see Husserl 1976/2014, §§40, 46; 1989 §§ 18f, 63).

constitutes this sense, and does so either by itself or in the whole motivational nexus pertaining to it and helping to make up its intentionality. (1974/1969, §94)[10]

Describing the givenness of the objects is tantamount to describing the constitution of the object. The constitution is the performance in which the object is (often merely passively) synthesized to what is given in experience.[11] And as we know (from our experience), our experiences are intertwined with our background knowledge, the results of empirical probing (such as trying by hand whether the stick half in water is bent or not), and empirical investigations, which all belong to the motivational nexus mentioned in the above quote. Hence, we understand the phenomenological and the empirical perspectives on what there is to be intertwined and complementary.[12]

The metaphysical neutrality of transcendental phenomenology means that the method does not add anything to or take anything away from its object.[13] For short, transcendental phenomenology is supposed to describe only what is given, and any explanatory metaphysical postulation or reduction is excluded from it.[14]

Even though the method itself is metaphysically neutral, the use of the method has metaphysical implications.[15] Since the method aims to describe the mode of being of the objects that are initially

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[10] In *Ideas I* Husserl discusses the experienced mode of being of objects and the kinds of evidence in §§136-138.
[11] Zahavi recaps 'constitution' "as a process that allows for manifestation and signification, that is, it must be understood as a process that permits that which is constituted to appear, unfold, articulate, and show itself as what it is" (2003, 73)
[12] The interdependency of the empirical and the phenomenological approach is explained in more detail in Zahavi (2017, Chapter 5).
[13] To be sure, in phenomenological reduction the entire world has been "suspended," but this means that nothing is lost, "every instance of worldly transcendence" is still there, but now viewed as *constituted*. (Husserl 1976/2014, §50). In other words, phenomenological reduction produces a change in the point of view on the world and everything in it; it does not take anything from it.
[14] This understanding of the metaphysical neutrality of phenomenology is clearly stated in the *Logical Investigations* (1984, 28-29/2001, 179), and it is also captured by Husserl's "principle of all principles" in *Ideas I* (Husserl 1976/2014, §24). A particularly nice passage witnessing Husserl's metaphysical neutrality toward abstract objects can be found in §22 of *Ideas I*: "In truth, everyone sees 'ideas,' 'essences,' and sees them, so to speak, all the time; everyone operates with ideas and essences in thinking – only from their epistemological 'standpoint' do they interpret those judgments away"((1976/2014, 48/40). The phenomenologist aims at capturing what "in truth" everyone assumes in their practices.
[15]Dan Zahavi has voiced a critique that Carr's and also Steven Crowell's interpretation leads to a "semantical" interpretation" of Husserl's phenomenology which makes it into an analysis of meaning that is not concerned with reality (2002, 110-111; 2017, 63-64, 101). On the present formulation of the phenomenological method, no such split is assumed, because the interwoven correlation of the natural and transcendental attitudes (to which Carr subscribes, too). Zahavi argues that because of its metaphysical implications transcendental phenomenology is not metaphysically neutral (2017, 63-76). The view advocated here agrees with both parties of this dispute in holding that the *method* is

encountered in the natural attitude, it describes the natural, that is, the unreflected, and non-philosophized metaphysical beliefs. To capture them, Husserl gives detailed descriptions of our natural attitude, which includes the common sense as well as the scientific attitudes of various disciplines together with what is given in them. By means of sense-investigation, as discussed above, Husserl includes an account of what is given in the scientific attitude as seen from practitioner's point of view. These descriptions yield descriptions of various "sub-worlds," or regions of being and hence it also results in the accounts of regional material and formal ontologies (Husserl 1976/2014, §9). Formal ontology treats what pertains to any object in general, and hence it includes formal mathematics.[16] The regional ontologies are ontologies of certain material region, such as of physical objects, mind, living organisms, etc. They also include the ontology of social entities, which Husserl discusses already from 1910 onwards (Husserl 1973b, 77-89; 98-104).

The role of the transcendental phenomenology is to clarify the conditions of possibility of these accounts and relate them to each other.[17] Husserl moves between the natural and the transcendental

metaphysically neutral but also that in its application metaphysical assumptions are revealed and thus it has metaphysical *implications*. These implications amount to "metaphysics" that is based on the analysis of the natural attitude and therefore they imply a rejection of those metaphysical views that are not based on it. Husserl's approach resembles that of Kant's in his rejection of naïve metaphysics but makes room for "synthetic a priori," critical metaphysics. This kind of understanding agrees with Husserl's claim in *Cartesian Meditations* that "phenomenology indeed excludes every naïve metaphysics that operates with absurd things in themselves, but does not exclude metaphysics as such" (1973a/1999, §64,182/156). This issue admittedly merits a paper on its own.

[16] In 1918, Husserl explained that formal ontology includes "pronouncements about objects in general, properties and relations in general, about sets in general, about what holds for sets in general with regard to their containing one another or being mutually exclusive, or what holds for numbers in general with regard to their different relationships grounding in the essence of numbers. Likewise, propositions about the relations between whole and part, about sequences and ordinals, and so forth. That is the field of formal ontology"(2019, §46b). In *Formal and Transcendental Logic* Husserl realizes that the ontology of pure mathematics cannot be identified with the ontology of what exists in the world. Hence, he concludes that "*[t]he aforesaid pure mathematics of non-contradiction*, in its detachment from logic as theory of science, *does not deserve to be called a formal ontology*. It is an ontology of pure judgments *as senses* and, more particularly, an ontology of the *forms* belonging to non-contradictory – and, in that sense, possible – senses: possible in distinct evidence" (1974/1969, §54b). After this, his interest in formal ontology as provided by pure mathematics fades, and he focuses on the formal ontology of the world.

[17] For example, in his 1917/18 lectures on *Logic and General Theory of Science* Husserl explains how in the realm of natural knowledge "analytic-formal ontology comes before synthetic-formal ontology, namely, the ontology of nature. It would break down into several separate disciplines, into the disciplines that explore the a priori of space and time, therefore geometry, chronology, and kinematics, on the other hand, into those disciplines that would correspond to Kantian 'pure' natural science, therefore, explore the a priori of spatiotemporal reality (matter)- except for pure space- and time-form" (Husserl 1996, 278/Husserl 2019, 294-295). This then takes Husserl to consider the categories in both Aristotelian and Kantian sense. It also takes Husserl to claim that there should be something like social ontology, the possible objections of the sociologists withstanding: "The idea of an a priori analysis of the collective spiritual life and its objective correlates is so far from the thoughts of sociologists and historians especially that probably just the contention that there could be anything of the kind and that [it] might be the necessary epistemological basis of all genuine social science would undoubtedly be declared by the sociologists to be mysticism or scholasticism" (Husserl 1996, 284-285/2019, 301).

attitude in a zig-zag fashion: his description of the natural attitude is already influenced by his transcendental phenomenological reflections, which again are reflections on what is given in the natural attitude. Similarly Husserl's discussion of sense-investigation and transcendental phenomenology as discussed above are intertwined. Thus his metaphysical view—the result of using the metaphysically neutral method—can be said to be the transcendentally informed account of what there is according to the natural attitude toward given region of being, or as in *Formal and Transcendental Logic*, toward the subject matter of mathematics as the mathematicians are directed to it.

When investigating mathematics, the phenomenologist tries to capture the mathematicians' view of the purpose of their practices. The results of this investigation are transcendentally clarified so that the practitioners' implicit presuppositions and the intended kinds of evidence, basic principles and concepts, are made explicit and clarified. The metaphysical neutrality of transcendental phenomenology means that the method does not add anything to or take anything away from its object. However, the results of using such a method are obviously metaphysical. For example, the method excludes the (metaphysical) approaches that do not agree with the practitioners' point of view. Hence intuitionist philosophical revisionism,[18] logicism, scientific naturalism, fictionalism, and so forth, are excluded. Furthermore, in its aim to spell out the practitioner's implicit metaphysical beliefs, for example, their common sense belief in the existence of the world (*doxa*), the method implies a kind of "metaphysics". Hence, structuralist, platonist, and constructivist elements can be found in the practice of mathematics, as described in *Formal and Transcendental Logic* (see Hartimo 2021). The upshot is that the phenomenological method, while ruling out some metaphysical views, leaves us with several open possibilities for a metaphysical approach to mathematics.

All in all, our aim is to show that Husserl manages to formulate a metaphysically neutral, yet critical, method with which to approach the practitioner's point of view. The method yields a critical and reflective analysis of mathematicians' aims, how well the mathematicians realize them, and what kind of reality they think they are tracking. Thus it promises to provide a rather satisfactory understanding of mathematical practice.

[18] Intuitionism is excluded in the sense of a philosophy-first revisionist approach. The practice of constructive mathematics is not excluded but taken at face value.

But this also raises a question, whether it is enough? The phenomenological method as characterized here might satisfy the *mathematicians* who seek to understand their experiences of their own subject matter. The analysis might also be enough for the phenomenologist interested in the givenness of mathematics, but as a *metaphysical* view, it is not entirely satisfactory. To sum: phenomenology is useful in accounting for mathematical practice, but it falls short in answering metaphysical questions about it (as the phenomenologist is not even interested in such questions). Hence, it is not clear what are the full metaphysical implications of the phenomenological method.

3. The need for further metaphysical considerations

This leads us to Leng's (2002, 8) suggestion that independent purely philosophical considerations may be needed in deciding which candidate – among the different practice-sensitive views of mathematics supported by the phenomenological approach – to back as the truth about mathematics. The phenomenological method applied to mathematics studies how mathematical objects are given to mathematicians when they practice mathematics, and thus approaches the ontology of mathematics from the practitioners' point of view. However, some questions cannot be answered just from within mathematical practice, as the following quotes illustrate:

> But what the philosopher is concerned with is, rather, to explain in what metaphysical sense, if any, mathematical objects exist, in a way that cannot even be discussed within ordinary mathematical parlance. (Feferman 2014, 90)

> Does mathematics have a subject matter, like physics, chemistry, or astronomy? Are mathematical claims true or false in the same sense? [...] The answers to these questions will not come from mathematics itself—which presents a wonderfully rich picture of mathematical things and their relations, but tells us nothing about the nature of their existence […] (Maddy 2007, 361)

Thus, we want answers also to traditional metaphysical and epistemological questions concerning the existence of mathematical objects and our knowledge of them. Here mathematical practice provides only part of the picture. While the phenomenological investigation of mathematics has metaphysical implications, its perspective that is internal to the experiences of these practices is not enough to flesh its metaphysical implications out in a manner that would satisfy a metaphysician.

In the interest of gaining a comprehensive understanding of the ontology of mathematics and its relations to other aspects of reality, we propose complementing the phenomenological method with an ontological account of mathematics. In order to give credence to the social nature of mathematical practice, we will take a social ontological approach that locates mathematical objects in the 'region of being' shared with social reality[19]. Curiously, this approach is still in many ways "Husserlian": Firstly, Husserl himself was among the first to discuss social ontology. Secondly, we claim that the project would still be phenomenologically justified in its metaphysical neutrality: it will not add any "spooks" or metaphysical "magic", nor is it reductionist about objects "that we see, all the time"(1976/2014, 48/40). Thirdly, Husserl himself saw mathematics as a historically developed social practice, [20] and finally, within this view we can conceptualize mathematics without forsaking its structuralist, constructivist, and platonistic aspects found in the phenomenological analysis of mathematical practice.

4. The social ontological approach: mathematical objects as social constructions

The metaphysical view we are after should offer an account of the ontology of mathematics that is justified by phenomenological understanding of the practice of mathematics. In accordance with the metaphysical neutrality that excludes "naïve" metaphysics, rather than making claims about "mathematical things in themselves", the aim is to give a metaphysical account of mathematical reality as it is encountered in practices. A suitable philosophical approach brings the lessons learned from the phenomenological and philosophical study of mathematical practice back into our account of mathematical ontology. The phenomenological approach to mathematical practice tells us that an appropriate ontology of mathematics depends on what mathematicians do and what their genuine goals are. Further, contemporary philosophy of mathematical practice has shown that

[19] Or as Szanto (2016, 147 fn. 5) puts it, the region of cultural, intentional achievements.

[20] It should be noted that Husserl's view about mathematical entities changes. Still in 1917, he held that mathematical objects are eternal and unchangeable (Husserl 2019, 35), but in *Formal and Transcendental Logic* (1929), where Husserl takes into account the socio-historical genesis of the phenomena such as mathematics, Husserl holds that the judgments made in mathematics are thought to be available to us at all times "as convictions lasting for us from the time of their first constitution" (1974/1969, §73). This latter formulation is in line with the social constructionist view developed later in the paper. However, Husserl does not really elaborate on this issue.

historical and social factors can affect how mathematics is practiced in different ways.[21] Consequently, the historical and social factors should be taken into account when answering ontological questions.

This is where the tools and conceptions from social ontology come in handy. Note that our focus is not on the ontology of social groups or collective intentionality,[22] but on the social ontology of *entities* that depend on shared practices – examples of which include money, universities, marriages, and so forth. Thus, for our purposes of applying social ontological tools to mathematical ontology, the idea of social construction is especially relevant. Accordingly, the social ontological approach that we take to mathematics is a form of *social constructionism*.[23] Our aim in this paper is not to defend a specific theory of social construction, but rather to explicate the benefits of taking such a view on mathematics.

The core idea of social constructionism is that mathematical objects[24] are social constructions in the sense that they are intended or unintended products of mathematical practices.[25] They are in some ways similar to more familiar social entities: Mathematical social constructions come into existence by humans acting and thinking in certain ways and they depend on mathematical practices for their existence; without humans and their practices of counting, calculating, formulating theories, and proving theorems, there would be no mathematical objects. Still, despite their dependence on practices, the objects and structures studied in mathematics are taken to genuinely exist. In a similar way to universities or money, mathematical social constructions are talked about and dealt with as existing things, without engaging in pretense. Additionally, social constructions are not concrete objects taking up space in the world, substantiating the prevalent view that mathematics concerns abstract entities.

[21] Carter (2019) offers an overview of different studies and strands from the field of the philosophy of mathematical practice.
[22] For an analysis of Husserl's take on collective intentionality, an important topic in contemporary social ontology, see Szanto (2016).
[23] Social constructionism about mathematics can be fleshed out in different ways. For example, Cole (2015) presents a detailed, Searlean take on mathematical social constructivism. For Cole, mathematical facets of reality exist as collectively recognized "institutional" entities, whose function is to represent other parts of reality. Hersh (1997) calls mathematical objects a distinct variety of social-cultural-historic objects, although does not offer a full-fledged philosophical account of their nature.
[24] By mathematical objects we mean the objects of mathematical study, and as noted already by Husserl, these include structures, like the natural number structure, and not only individual objects.
[25] Following the definition of social construction from Haslanger (1995, 97).

What differentiates mathematical social constructions from other social entities is the fact that the practices of formulating theories, solving problems, justifying new methods or axioms, and the central practice of giving proofs are highly constrained in various ways, and consequently, so are the features of the objects that depend on these practices. Some important factors constraining mathematical practices are (1) normative constraints, such as the goals of non-contradiction and truth identified by Husserl, (2) inter-theoretical relations, like the systematic links between mathematical practices that restrict the admissible and constrain new mathematical developments (Ferreirós 2016)[26], (3) biological constraints, in the form of evolutionarily developed cognitive abilities that form the basis of our arithmetical, geometrical or logical knowledge (Pantsar 2021), and (4) constraints placed by the structure of the physical world, such as the ways we can interact with our environment or empirical applicability.

Crucially, although mathematical objects – as social constructions – depend on human activities and thus did not exist before humankind[27], some of the constraints are independent of the thoughts and activities of humans. As a result, even though they are socially constructed, mathematical objects can have objective features.[28] The constraints also account for the validity, or seeming necessity, of mathematical facts, although their validity varies somewhat due to the different degrees to which they are constrained. The idea is that while elementary mathematics is presumably fully constrained by external factors (such as how physical objects can be manipulated), and that whether 2+2=4 is not up to us, the merely socially constructed element is greater about the higher mathematics. Thus, for questions like whether the axiom of choice or judgments about higher cardinalities should be accepted, the mathematical community has a greater degree of latitude, and social factors can play a part in reaching a consensus. However, the determinations of the mathematical community on such questions are still to some degree constrained by the links to previous mathematical practices and by the normative goals of

[26] For example, Ferreirós (2016, ch. 8) looks at the ways the development of real numbers was constrained by preceding practices. For one, irrational numbers became accepted because methods of calculating decimal fractions were available. Secondly, the notion of real numbers forming a complete and continuous system of numbers was inherited from the classical idea of a geometric line. Thirdly, the modern way to define real numbers arithmetically from the natural numbers requires a practice of viewing natural and rational numbers as complete totalities.

[27] Or possibly, before any other cognitive agents engaging in some mathematical activities.

[28] For instance, Cole (2015) claims that the semantic objectivity of mathematical statements is due to the features of the socially constructed mathematical entities being constrained by the ontologically objective facets of reality they represent. Rytilä (2021) argues that Cole’s representational explanation is not sufficient, and that explaining objectivity requires elaborating also on other constraints placed on social construction of mathematical entities.

mathematics. For instance, the “Husserlian” goal of non-contradiction ensures that mathematicians could not collectively decide to accept the axiom of choice but reject the well-ordering theorem (as these have been shown to be equivalent). In sum, the point is that the constraints placed on mathematical practices can account for the experiences – looking from inside the practice – that for the most part mathematics is decidedly not up to us and even necessary, even though the objects of mathematics are social constructions.

The idea of mathematical objects existing as social constructions accords well with the results reached with the phenomenological approach to mathematical practice. To see how, let us start by taking stock of the key lessons that can be learned about ontology with the phenomenological method. Firstly, mathematics is historically developed and varied in its methodologies. The implication of this fact is that what is given to mathematicians is somewhat context-sensitive. For instance, classical and constructivist mathematicians experience to some extent different objects and in different ways. Thus, the phenomenological approach to mathematics gives an essentially pluralistic view of mathematical ontology. Secondly, mathematicians experience the objects of their study as transcendent, i.e. external, and abstract objects. The implication is that the mathematical reality mathematicians deal with in their activities has some sense of independence to it.

Social constructionism as a metaphysical view incorporates these lessons by offering an account of mathematical ontology that acknowledges both the significance of human practices and historical and social factors, and the experience that mathematics is about abstract things that exist externally to the individual mathematician. Moreover, the view not only agrees with the phenomenological approach but the metaphysical account it provides takes the lessons a step further.

Starting with the lesson about the context-sensitivity of mathematical ontology, social constructionism takes mathematical objects to be products of socially shared and accepted human practices of mathematics. Thus, the influence the particular social and historical contexts can have on the development of mathematical practices, and subsequently on the subject matter of mathematics, is built into the metaphysics. On this point social constructionism helps in “cashing out” the teachings of transcendental phenomenology regarding ontology of mathematics

Social constructionism also shares a similar pluralistic approach to ontology as is found by applying the phenomenological method. According to social constructionism, the different practices – like those of classical and constructive mathematics, or Euclidian geometry and modern axiomatic set theory – can give rise to different kinds of objects. However, looking from a philosophical viewpoint, outside of particular mathematical practices, all the various mathematical objects – be they platonistically understood objects, explicitly constructed objects, or formal structures – have a similar metaphysical nature, and they occupy the same region of reality. The point is that although the features of the objects generated by the different practices may vary insofar as the practices do, all the different objects exist as intersubjectively shared, socially constructed objects. This is also the case for social constructions in general, as the part of reality that depends on social practices for its existence contains a wide range of different kinds of entities. The social world also contains entities that overlap in terms of their purpose and features despite existing separately and having significant differences. For instance, there are many different legal systems, with different laws and law-making procedures, but with the same purpose and manner of existing. In a similar way, from the social constructionist viewpoint differing mathematical ontologies have distinct features but the same kind of existence, metaphysically speaking. Consequently, the social ontological approach is able to provide a unified metaphysical picture of pluralistic mathematical ontologies.

Moving to the second lesson, social constructionism substantiates the phenomenological finding that mathematical objects are experienced as externally existing, abstract objects, because the view considers mathematical objects to exist as culturally shared, abstract entities. Borrowing a phrase from Hersh (1997, 16), "[o]nce created and communicated, mathematical objects are *there*. They detach from their originator and become part of human culture." After being introduced, mathematical objects are studied as external objects. Furthermore, they can have features that are difficult to discover. On this point mathematical social constructions are not qualitatively different from social entities. Thomasson (2003) points out that once some part of the social world is constructed, there will be all sorts of patterns and features within it, and in many cases the participants may not have any awareness of or beliefs about them. While they ultimately depend on human practices, they can exist without anyone having any thoughts about the patterns and features as such. An example would be a recession; it is a state the economy can be in without anyone having beliefs about it. And after studying the economy, social scientists can make the discovery that there in fact is a recession. Similarly, mathematical social constructions have features and relations that are not immediately apparent to mathematicians but can be discovered through mathematical research.

The benefit of the social constructionist view is that we can assume some kind of independent existence for mathematical objects, as well take them to have some unknown features, while still maintaining that the objects are not completely separate from mathematical practice. The sense of independence comes from two sources. First, because the practices are shared, the objects generated by them have an intersubjective reality, meaning they do not depend on any one person's consciousness.[29] Second, because of the various constraints, mathematical social construction results in objects whose features are largely not up to us, even to the extent that they seem necessary. As a result, they can track objective facts of mathematical depth, as Maddy (2011) puts it. These facts of mathematical depth can be seen as the historical facts of the fruitful use of particular notions, statements, and theories in mathematical practice, as is suggested by Imocrante (2015). In other words, socially constructed mathematical objects are shaped by previously successful forms of mathematical practice.

So, according to this social constructionist view, while mathematical objects ultimately depend on the participants of mathematical practices, they are not dependent on what you or I happen to think about them but rather on constrained and intersubjectively accepted mathematical practices. Hence, for the individual mathematician, the existence and features of a particular mathematical object is as much a matter of their own thoughts and wishes as those of any objective thing. Understanding mathematical objects from the social ontological viewpoint thus allows us to treat them as existing and in some sense objective things in good conscience.

The result from the first two lessons combined is that a social constructionist account of mathematical ontology can validate contrasting aspects of the experience of practicing mathematics, which the phenomenological approach studies. The view endorses both the sense that mathematicians are free to create new objects to attain their goals in particular contexts and the sense that the mathematical reality is given to the mathematicians with a sense of

[29] This intersubjectivity constitutes a kind of objectivity, as is also seen in Husserl's phenomenology. Szanto (2016) describes how, in Husserl's view, we always already stand in interpersonal and social relations to one another as inhabitants of a shared lifeworld. Thus, the intentional experiences of individuals are imbued with a "we-perspective", resulting in a commonality of the experienced reality.

independence.[30] This is because mathematical social constructions are human creations but at the same time have some independence of the thoughts and beliefs of mathematicians. Thus, the social ontological approach offers a way to flesh out the metaphysical implications of the phenomenology of mathematical practice.

In sum, the social ontological approach to mathematical reality shares much with the phenomenological approach and is able to do justice to the phenomenology of mathematical practice. However, while Husserl, too, acknowledges the intersubjectivity and cultural, or social, aspects of mathematical practice, he does not properly explore their implications for mathematical ontology. By doing just so, the social ontological approach is able to provide an account of the ontology of mathematics that is phenomenologically justified and takes the practice-based approach to mathematical ontology even further than what Husserl did.

5. Are mathematicians (the worst) magicians?

We argued above that as a *metaphysical* view of mathematics, the social ontological approach goes beyond the limitations of the phenomenological approach by considering how to build the ontology out of the practice. However, in order to be justified by the metaphysically neutral phenomenological approach, the social ontological account should not postulate new objects any which way, or on purely philosophical grounds. This raises a worry about the legitimacy of the objects' existence: Is it not unnecessary and even suspicious to posit existing mathematical objects on top of the practices? Are mathematicians just conjuring the objects of their study into being, as if by magic?

First of all, taking mathematical objects to exist is not wholly unnecessary, because it allows us to understand mathematical language literally and take the way mathematicians speak about their subject matter seriously. As Shapiro (1993, 458) notes: "Mathematics is, after all, a dignified and vital endeavour, and we would like to think that mathematicians mean what they say. This is to take it "at face value"." The benefit of accepting face value readings of mathematics is that it saves us from claiming that both experts and laypeople are wrong when they make assertions about, say, numbers, that are taken to be obviously true (see Linnebo 2018, 9).

[30] For a similar argument, see Cole (2009).

However, granting the usefulness of taking mathematical objects to exist still leaves the worry about their existence being some kind of magic trick. This is because the socially constructed mathematical objects have a kind of "lightweight" existence, compared to the existence of concrete things or the activities actually performed by mathematicians in the physical world. The idea is that the existence of mathematical objects does not require substantially much more from the world than the existence of the appropriately constrained practices. As such, social constructionism has some similarities with metaontological minimalism of Linnebo (2018) – the view that there are "thin" objects whose existence does not make a (further) substantial demand on the world [31] – and can face the same 'magic objection':

> Metaontological minimalism can come across as a piece of philosophical magic that aspires to conjure up something out of nothing – or, in the relative case, to conjure up more out of less. (Linnebo 2018, 5)

Here again social ontology can be of help since similar worries have been raised about social entities as well. Searle (1995, 45) describes the worry as "[o]ur sense that there is an element of magic, a conjuring trick, a sleight of hand in the creation of institutional facts out of brute facts […]". The conjuring trick that Searle alludes to is the idea of merely speaking or defining things into existence, such as bringing a climate committee into existence by deciding and declaring to form one. The gist of the magic objection is to question the possibility of creating new entities just by speaking some words aloud, as if we were wizards or magicians.

Amie Thomasson (2019) argues that this is a misguided worry. If there is magic involved in generating social entities, like a climate committee or a corporation, it is such poor magic as to not be magic at all.

"Consider the worst magic trick in the world:

- Nothing up my sleeves… I'll just put this right glove and this left glove in a hat and… Shazam! A pair of gloves emerges!" (Thomasson 2019, 4831)

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[31] As an example, the existence of the set of all books in the office requires little or nothing else beyond the existence of the books, and thus the object 'set of books' is "thin" relative to the books themselves, which are "thick" objects (Linnebo 2018, 4). For Linnebo, mathematical objects are thin objects that are obtained through abstraction principles. A classic example concerns lines and directions: $l_1 \parallel l_2 \Leftrightarrow d(l_1) = d(l_2)$. The point is that the existence of directions, which are thin objects, does not require anything more than the parallelism of the appropriate lines.

In her easy ontology -approach, all it takes for a certain object to exist is for the term's application conditions to be fulfilled. There is nothing more mysterious required for a corporation to exist than the relevant papers having been filed, just as there is nothing more needed for a pair of gloves to exist than the existence of a left glove and a right glove. Only the worst magicians in the world would treat the creation of these commonplace things as magic tricks. Moreover, it is important to note that the application conditions differ from object to object. For the climate committee, just a declaration and appointing members may suffice for its existence, but for a law to exist, a vote by members of parliament or other legal proceedings are required, and still in other cases, some underlying physical or historical facts may be needed (Thomasson 2019, 4831). In most cases, there is more needed for a social entity or an object to exist than merely saying some relevant "magic words", which takes out some of the force from the magic objection.

Even leaving Thomasson's approach to the side, a key reason to assume that social entities genuinely exist is that we deal with social entities all the time in ordinary life as well as in social sciences. Denying the existence of things like marriages, climate committees, universities, recessions, and so forth makes little sense in those contexts. On this point social ontology is well in line with the phenomenological method, since it too, takes experience as is, without reducing experienced objects away on metaphysical grounds. Furthermore, the general view among social ontologists is that social entities are not a product of any mysterious or magic-like effect. To take an example, at a wedding, a new social entity – a marriage between particular people – is created. But rather than assuming the officiant to have a magic-like power, philosophers and social scientists can explore what actually makes this new entity come into being, whether it is laws concerning marriage, signing relevant papers, a collective recognition of the new status of the couple, or something else. Indeed, one aim of social ontology is to explain how social entities and phenomena come about.

Since from the social constructionist viewpoint, mathematical objects have similarities with social entities, and lack some characteristics traditionally attributed to them, such as timeless or necessary existence,[32] the same reasons apply in the context of mathematics. For example, Carter (2004)

[32] In the social constructionist account as described in this paper, the mathematical objects themselves are not timeless or necessary, although some of the constraints that shape their features may be. At the same time, within the practice, the mathematicians may experience them as timeless, making the timelessness of the objects a matter of a point of view. This is a poignant example showing in what sense social constructionism goes further than the phenomenology of mathematical practice

makes the case that because we can successfully speak about mathematical objects, they are entitled to some kind of existence[33]. Further, instead of regarding this existence as magic-like, philosophers of mathematical practice can study which objects mathematician do successfully speak about or create, and what the conditions and reasons for this success in the mathematical practice are. If we are entitled to treat social entities as existing things without assuming any magic, we are similarly entitled to treat practice-dependent mathematical objects as existing.

Where socially constructed mathematical objects differ from social entities is in the conditions that need to be met for the objects to exist. Thomasson (2003) notes that many constructed social kinds have no 'deep' application conditions that cannot be met merely through social agreement. This is not the case for mathematical objects. Mathematicians do not simply decide in a conference to create a new mathematical object. As noted above, mathematical practices are constrained in various ways. Whatever the relevant constraints may be in each case, the point is that they impose further conditions for the existence of mathematical objects, in addition to collective agreement among the mathematical community. As such, the conditions for the existence of socially constructed mathematical objects are typically more demanding than for many social entities.

The upshot is that if we can do without magic in social ontology, we have even less reason to assume any metaphysical magic in practice-dependent mathematical ontology. Moreover, viewed from the social ontological viewpoint, mathematical objects are grounded on actual practices and experiences of mathematicians. Thus, socially constructed mathematical objects can reasonably be taken to exist without violating the metaphysical neutrality of the phenomenological approach, and with no magic.

6. Conclusion: Phenomenology and social ontology as complementing approaches to mathematics

In conclusion, our aim in this paper is to show that phenomenology and social ontology offer complementing philosophical approaches to mathematical practice in a systematically interesting way. Husserl's phenomenological method, understood as a combination of sense-investigation and transcendental reflection, aims to elucidate and clarify the practitioner's point of view. Applied to

[33] More specifically, Carter (2004, 257) argues that mathematical objects exist as a kind of abstract objects – meaning human constructions or creations – after they have been introduced.

mathematics, the phenomenological method provides a critical and reflective analysis of mathematicians' aims, how well the mathematicians realize them, and what kind of reality they think they are tracking. Thus, the phenomenological approach to mathematics yields a detailed understanding of mathematical practice.

However, the Husserlian phenomenological method is metaphysically neutral, and hence implies, but does not provide full-fledged answers to some philosophically, even if not phenomenologically, relevant questions on the metaphysical nature of mathematical objects. This can be done by shifting to a social ontological viewpoint to mathematics. The social ontological approach as described here takes mathematical objects to be social constructions, that is, products of shared mathematical practices. By building the ontology out of mathematical practice, the social ontological approach offers a way to flesh out the metaphysical implications of the phenomenological method.

Furthermore, the social constructionist account gives legitimacy to the existence of practice-dependent mathematical objects, without violating the requirements set by the metaphysical neutrality of phenomenology. Due to their intersubjective nature and the various constraints placed on mathematical practice, the conditions for the existence of mathematical objects are typically more demanding than for social entities, which we commonly take to exist and successfully talk about. Thus, there is no need to assume any metaphysical magic to be involved in the social construction of mathematical ontology.

But while social constructionism offers an account of the metaphysical nature of the subject matter of mathematics, it lacks a detailed analysis of *mathematical* practice. This is needed to give an accurate account of the conditions and constraints placed on social construction of mathematical ontology. Fortunately, such an analysis can be offered precisely by phenomenological reflection of mathematics. Together, the phenomenological method and the social ontological approach form a comprehensive philosophy of mathematical practice.

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